Physics 443 Homework #1
Due Thursday, October 9, 2008

1.) Peskin & Schroeder, problem 2.1

2.) Peskin & Schroeder, problem 2.2

3.) Peskin & Schroeder, problem 2.3

4.) The classical limit of a harmonic oscillator can be described in terms of *coherent states*

$$|\alpha\rangle = \exp\left[\alpha a^\dagger\right]\ |0\rangle\ \ .$$

Proceeding similarly for the Fourier modes of the quantum Klein-Gordon field,

$$\begin{aligned}|f\rangle =& N_f\ \exp\left[i\int\frac{d^3p}{(2\pi)^3}f(\mathbf{p})a^\dagger_{\mathbf{p}}\right]\ |0\rangle\\ N_f =& \exp\left[-\tfrac{1}{2}\int\frac{d^3p}{(2\pi)^3}|f(\mathbf{p})|^2\right]\ \ .\end{aligned}$$

(a) Evaluate the expectation value of the field operator

$$\langle f|\,\phi(x)\,|f\rangle\ \ ,$$

and show that it satisfies the Klein-Gordon equation.

(b) Evaluate the relative mean square fluctuation of the occupation number of the mode with momentum $\mathbf{p}$, and the relative mean square fluctuation in the total energy

$$\frac{\langle\hat{n}^2_{\mathbf{p}}\rangle - \langle\hat{n}_{\mathbf{p}}\rangle^2}{\langle\hat{n}_{\mathbf{p}}\rangle^2}$$
$$\frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2}\ \ .$$

Is either of these a good measure of the degree to which the field is classical? Justify your answer.

(c) Take $\Delta(x-y) = \langle 0|\,\phi(\mathbf{x})\phi(\mathbf{y})\,|0\rangle$ (equal times) as a measure of the fluctuations or correlations of the field amplitude. Use your result for problem 2.3 (P&S) to evaluate this quantity. What is the meaning of the divergence as $\mathbf{x}\to\mathbf{y}$?