Physics 443 Homework #4
Due Tuesday, November 4, 2003

1.) Peskin and Schroeder 3.5a-b (not c)

2.) Among the uses of Grassmann numbers is to geometrize the supersymmetry between bosons and fermions. Consider a space parametrized by the ordinary spacetime coordinates x^μ, and also the Grassmann coordinates θ^α (anticommuting numbers that transform as a Weyl (chiral, or (1/2,0)) spinor under Lorentz transformations) as well as their conjugate ((0,1/2) or antichiral) coordinates $\bar{\theta}^{\dot{\alpha}}$.

a) Show that the derivative operators

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$
$$\bar{Q}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

satisfy the algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$
$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

Thus the Q's are like the 'square root' of a translation. Show similarly that the derivative operators

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$
$$\bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

anticommute with the Q's.

b) Let's organize the fields ϕ, χ_α, F of the previous problem (P&S 3.5) into a *superfield.* To see the structure, without all the notational baggage of 3+1 dimensional spinors, work in 1+1 dimensional spacetime and use light-cone coordinates $x_\pm = t \pm x$ and their Grassmann superpartners $\theta_\pm$. The superderivatives become

$$D_\pm = \frac{\partial}{\partial\theta^\pm} + i\theta^\pm\partial_\pm$$

Consider a 1+1d superfield

$$\Phi(x_\pm, \theta_\pm) = \phi(x) + \theta^\pm\chi_\pm(x) + \theta^+\theta^- F(x)$$

Because the θ's are anticommuting parameters, the expansion terminates at finite order. Show that the supersymmetry transformations

$$\delta\phi = \epsilon^\pm\chi_\pm$$
$$\delta\chi_\pm = \epsilon^\mp F - i\epsilon^\pm\partial_\pm\phi$$
$$\delta F = -i\epsilon^\pm\partial_\pm\chi_\mp$$

can then be written

$$\delta\Phi = [\epsilon^{\pm} Q_{\pm}, \Phi] \ .$$

Show that the 1+1d version of the Lagrangian of the first problem (including part (c)) can then be written in a manifestly supersymmetric way as

$$\mathcal{L} = \int d^2x d^2\theta \ [D_+ \Phi D_- \Phi + W(\Phi)]$$

Thus supersymmetric theories can be regarded as field theories on a parameter space including both commuting, 'bosonic' (x^μ) and anticommuting 'fermionic' (θ_α) coordinates. The usual notion of Poincare symmetry (Lorentz transformations together with translations) is extended to the larger group of Lorentz transformations, translations, and supertranslations (implemented by D_α, $\bar{D}_{\dot{\alpha}}$).

It is currently thought that supersymmetry is an approximate symmetry of nature above energy scales of about 1 TeV.

3.) Peskin and Schroeder 9.2d

Those who are looking for a little more reading on supersymmetry, Grassmann numbers, etc., may consult

J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd ed., Princeton U. Press (1992).

B. DeWitt, *Supermanifolds*, Cambridge U. Press (1992).

F.A. Berezin, *Mathematical Base of Supersymmetrical Field Theories* Yad. Fiz. **29** (1979) 1670-1687 (Russian); translated in Sov. J. Nucl. Phys.