Physics 443 Homework #5
Due Tuesday, November 18, 2008

1.) Peskin and Schroeder 4.1

2.) Peskin and Schroeder 4.2

3.) *Aspects of scaling behavior in QFT:*

a) As discussed in class, massless free scalar field theory is invariant under scale transformations, if we assign the appropriate scale dimension to the field ϕ. The action of dilations on coordinates is implemented by the operator $D = ix^\mu \partial_\mu$. Find the commutation relations of this operator with the operators that implement the Poincaré algebra of translations $P_\mu = i\partial_\mu$ and Lorentz transformations $J_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$. Find the canonical Noether current and Noether charge for this symmetry in the free field theory, and find the commutation relations of the charge with the field ϕ to show that it implements the infinitesimal form of the symmetry on the fields. Check also that this current has the right commutation relations with the field theory Hamiltonian and momentum generators.

b) The sound waves in a solid obey a wave equation, which at low enough wavelengths/energies is described by the action (for simplicity, consider one spatial dimension)

$$S_0 = \int dtdx[\partial_t \phi \partial_t \phi - c_s^2 \partial_x \phi \partial_x \phi]$$

where c_s is the speed of sound in the material. The field ϕ describes the distortion of the lattice away from equilibrium. Model this situation by a lattice of atoms (mass m, lattice spacing a) interacting through harmonic nearest-neighbor forces (spring constant k). Find the leading term in the action which corrects the above kinetic energy in a power series in $a \times$ *derivatives* (this can be done for instance by expanding the exact dispersion relation for the chain of masses and springs). Show that this correction term is an *irrelevant* perturbation of the action, so that all traces of the lattice structure disappear in the continuum limit $a \to 0$ (*e.g.* all traces of the structure of interactions at the atomic level, such as a general potential $V(r)$ between the atoms rather than harmonic forces, disappear, being summarized in the constant c_s – and the effective action S_0 is independent of the cutoff a). Estimate the momentum scale at which the irrelevant corrections amount to 10% of the total energy of a phonon.

(In three dimensions, we have to generalize the gradient kinetic energy to

$$S_0 = \int dtd^3x[\partial_t \phi_i \partial_t \phi_i - D^{ijkl} \partial_i \phi_j \partial_k \phi_l]$$

where ϕ_i, $i = 1, 2, 3$, describe the displacements in the three spatial directions. Some features of the lattice structure survive in the effective action in that the symmetry properties of D^{ijkl} reflect the crystallographic symmetries of the lattice; for instance the speed of sound in different directions, or for longitudinally versus transversely polarized phonons, typically differ. However these features are orientational in nature; the effective action is still independent of the lattice spacing at leading order.)