This is a combined reading assignment/problem set which you should work on during the first week and a half of classes when I will be away. The first class will meet on Jan 16. Make-up lectures will be scheduled later in the quarter. I will be back on Jan 13 and will respond to email questions about the problem set on that date. I will also be available on Monday, Jan 15 including office hours at the same time as last quarter.

Reading: Read about the separation of variables for three-dimensional systems with spherical symmetry and about the solution to the radial Schrodinger equation for various systems including the Hydrogen atom. This material is covered in Sections 12.6,13.1,13.2,13.3,13.4 of Shankar. You should then work the first set of problems. Following this, read about time independent perturbation theory. This is covered in Sakurai secs. 5.1 and 5.2 and in Shankar, Chapter 17. You should be familiar with the form of the first and second order expressions for the change in energy due to a perturbation and the formalism of degenerate perturbation theory. After studying this material you should work the second set of problems. Much of this material should be review as it is standard material usually covered in undergraduate quantum mechanics courses. The other topics I am planning to cover this quarter are the path integral formalism, particles in electromagnetic fields and the role of gauge invariance, Berry's phase and the adiabatic approximation, and measurement theory. If time permits we will do one of the other topics listed in the Ph 341 course syllabus.

Problems on Spherical Potentials

1. This problem is taken from the 1995 candidacy exam. A particle of mass m moves in a spherically symmetric "δ-shell" potential

$$V(r) = -\frac{\hbar^2}{2Lm}\delta(r-a)$$

with L a parameter with dimensions of length and the delta function is defined by $\int F(r)\delta(r-a)dr = F(a)$ if the integration region includes the point $r = a$ and is 0 otherwise.

   (a) Derive the boundary conditions obeyed by the wave function at $r = a$.

   (b) For a special choice of L there will be an angular momentum $\ell = 0$ bound state near zero energy. Find the bound state energy as a function of L near this special choice.

2. Consider a modified Coulomb potential

$$V(r) = -\frac{Ze^2}{r}\left(1 + b\frac{a_0}{r}\right)$$

Here Z, e are as usual, a_0 is the Bohr radius, and we will assume that $b << 1$ so that the effect of the modification is small except when r is much less than a_0. Find the energy eigenvalues for orbital angular momentum $l = 0$ for a particle moving in this potential. (Hint: use a power series analysis as in Shankar's discussion of the radial equation for the hydrogen atom).

3. Consider a particle of mass m in a spherical potential with $V(r) = -V_0$ for $r \leq a$ and $V(r) = 0$ for $r > a$ with V_0 a positive constant.

   (a) Show that for V_0 sufficiently small this problem has no angular momentum $l = 0$ bound state.

   (b) Now suppose that V_0 is such that several bound states exist. *Estimate* the maximum angular momentum l for which you expect a bound state to exist.

Problems on Time Independent Perturbation Theory

4.

   (a) Calculate the first order energy shift in the ground state of the one-dimensional harmonic oscillator when the perturbation

$$V = \lambda x^4$$

   is added to

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2.$$

   (b) Now consider the perturbation

$$V = \lambda x^3$$

   added to H_0. Calculate the energy shift of the ground state to second order in λ.

5. Consider a perturbation of a two-dimensional harmonic oscillator with

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m\omega^2}{2}(x^2 + y^2) + \delta xy(x^2 + y^2).$$

   Compute the corrections to the energies of the three lowest energy levels to first order in δ.

6. Two identical spin 1/2 fermions are placed in a cubical box of volume V. In addition to being confined in the box there is a short range interaction between the two fermions which we approximate as a delta function interaction

$$V(\vec{r}_1 - \vec{r}_2) = -\frac{4\pi a^3 V_0}{3} \delta^3(\vec{r}_1 - \vec{r}_2)$$

   where a represents the effective range of the interaction and V_0 its strength. Calculate the ground state energy of this system to first order in V_0. Be sure to take Fermi statistics into account.