Problem Set 1

Physics 330

Due October 5

Some abbreviations: A - Arfken & Weber, MW - Mathews & Walker.

1. (A8.2.6)
Show that the substitution $u = y/x$ makes the equation,

$$y' = g(y/x), \tag{1}$$

separable in u and x.

2. (A8.2.17)
Solve the first-order equation,

$$y' + p(x)y = q(x), \tag{2}$$

by assuming $y(x) = u(x)v(x)$ where $v(x)$ is the solution to the homogeneous equation ($q = 0$). This is the method of variation of parameters due to Lagrange, and a simpler version of the osculating parameter approach discussed in class.

3. (A8.6.25)
The most general solution for the equation,

$$y'' + P(x)y' + Q(x)y = F(x), \tag{3}$$

takes the form:

$$y(x) = y_1(x) + y_2(x) + y_p(x). \tag{4}$$

In this expression, y_1 and y_2 are solutions to the homogeneous equation. If the Wronskian $W\{y_1, y_2\}$ is given by,

$$W\{y_1, y_2\} = y_1 y_2' - y_1' y_2,$$

show that

$$y_p = y_2 \int \frac{y_1 F}{W\{y_1, y_2\}} - y_1 \int \frac{y_2 F}{W\{y_1, y_2\}}.$$

Hint: try to use a similar technique to the one just used in the previous problem.

4. Show that $y'' = f(y)$ can be integrated immediately if both sides are multiplied by y'.

For problems $\mathbf{5-14}$, do MW $1-1$ through $1-10$. These problems are given below. Find the general solutions to the following equations:

5. $x^2 y' + y^2 = xyy'$.

6. $y' = \frac{x\sqrt{1+y^2}}{y\sqrt{1+x^2}}$.

7. $y' = \frac{a^2}{(x+y)^2}$.

8. $y' + y\cos(x) = \frac{1}{2}\sin(2x)$.

9. $(1-x^2)y' - xy = xy^2$.

10. $2x^3y' = 1 + \sqrt{1+4x^2y}$.

11. $y'' + (y')^2 + 1 = 0$.

12. $y'' = e^y$.

13. $x(1-x)y'' + 4y' + 2y = 0$.

14. $(1-x)y^2dx - x^3dy = 0$.