Problem Set 4

Physics 330

Due November 9

Some abbreviations: A - Arfken & Weber, MW - Mathews & Walker.

1-4 More fun with contour integration. Do problems MW 3-10, 3-13, 3-15, 3-22. Do the integrals by contour integration:

1. $\int_0^\infty \frac{dx}{1+x^4}$.

2. $\int \frac{d^3x}{(a^2+r^2)^3}$.

3. $\int_0^\infty \frac{x\,dx}{1+x^5}$.

4. $\int_0^\infty \frac{\sin(x)\,dx}{x(a^2+x^2)}$.

5. Clausen's paradox (taken from unpublished notes by Greg Moore). Consider the following argument:

(i) $e^{2\pi in} = 1$ for all integers n.
(ii) $e = e \cdot 1 = e \cdot e^{2\pi in} = e^{1+2\pi in}$.
(iii) If $x = y$ then $x^z = y^z$ therefore $e = e^{1+2\pi in}$ implies:

$$e^{1+2\pi in} = \left(e^{1+2\pi in}\right)^{1+2\pi in}. \tag{1}$$

(iv) So,

$$e = e^{(1+2\pi in)^2} = e^{1+4\pi in-4\pi^2n^2}, \tag{2}$$

(v) but $e^{1+4\pi in-4\pi^2n^2} = e \cdot e^{-4\pi n^2}$.
(vi) Therefore, $1 = e^{-4\pi n^2}$ for all integers n!

Question: the 'result' is clearly wrong. Where is the wrong step?

6. MW 3-31.

(a) Consider the contour integral $\oint f(z)\mathrm{ctn}(\pi z)dz$ around a suitable large contour, and thereby obtain a formula for the sum

$$\sum_{n=-\infty}^{\infty} f(n). \tag{3}$$

(b) Evaluate

$$g(a) = \sum_{n=-\infty}^{\infty} \frac{1}{n^2+a^2}. \tag{4}$$