

Problem Set 2

Physics 445

Due May 12

Some abbreviations: P&S - Peskin & Schroeder

1. Try working with a different non-covariant gauge choice. P&S 16.1.
2. Consider the QED Lagrangian with the gauge choice $\partial_\mu A^\mu = 0$ and retain the associated ghost terms. Verify that the Lagrangian is invariant under the BRST transformation,

$$\delta A_\mu = \epsilon \partial_\mu c, \quad \delta \psi = 0, \quad \delta c = 0, \quad \delta \bar{c} = \epsilon \partial_\mu A^\mu.$$

Construct the associated Noether current and conserved charge. Canonically quantize the theory and express the conserved charge, Q , in terms of creation and annihilation operators for the gauge and ghost fields. Verify the (anti-)commutation relations between Q and the annihilation/creation operators given in lecture.

3. The usefulness of differential forms: let's consider a p -form

$$\omega = \frac{1}{p!} \omega_{i_1, \dots, i_p} dx^{i_1} \cdots dx^{i_p} \equiv \omega_{|i_1, \dots, i_p|} dx^{i_1} \cdots dx^{i_p}$$

where vertical bars around a set of indices indicate they are summed only over $i_1 < i_2 < \cdots < i_p$. Define the dual form, $*\omega$, which is an $n - p$ form in n dimensions as the form with components

$$(*\omega)_{k_1, \dots, k_{n-p}} = \omega_{|i_1, \dots, i_p|} \epsilon_{i_1, \dots, i_p, k_1, \dots, k_{n-p}}.$$

Here $\epsilon_{i_1, \dots, i_n}$ is the totally anti-symmetric Levi-Civita tensor. In QED, define a 1-form gauge potential $A = A_\mu dx^\mu$ with a 2-form field strength $F = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu$.

- (i) Express F in terms of A using the exterior derivative d .
- (ii) Write Maxwell's equations in terms of d , F , and $*F$.
- (iii) Repeat this exercise for non-abelian gauge theory. Define a Lie-algebra valued 1-form $A_\mu^a t^a dx^\mu$ and write F in terms of d and A . Express the equations of motion in terms of forms.

4. To get a feel for β -functions, do P&S 16.2.