

Problem Set 3

Physics 445

Due May 22

Some abbreviations: P&S - Peskin & Schroeder

1. In typical quantum field theories, we expect generic couplings to be renormalized. There are special cases where a symmetry can protect a coupling from renormalization. The most important of these symmetries is supersymmetry. This symmetry is fermionic, much like the BRST symmetry we studied earlier.

To learn about supersymmetry, consider the following Lagrangian:

$$L = \phi^* \partial_\mu \partial^\mu \phi - |\lambda|^2 |\phi|^4 - \{\lambda \psi \psi \phi + \text{c.c.}\} - i \psi \sigma^\mu (\partial_\mu \bar{\psi}). \quad (1)$$

Here ϕ is a complex scalar field, ψ_α is a two component complex Weyl fermion, and the coupling λ is complex. The $\sigma^\mu_{\alpha\dot{\alpha}}$ are the Pauli matrices with $\sigma^0 = -\mathbf{1}$. The notation $\psi\psi = \psi^\alpha \psi_\alpha = -\psi_\alpha \psi^\alpha$ where the spinor index is raised and lowered with an epsilon tensor, $\epsilon^{\alpha\beta}$ (normalize $\epsilon^{12} = 1$). While the undotted index ψ^α indicates a field transforming in the $(1/2, 0)$ representation of Lorentz, the dotted index $\psi^{\dot{\alpha}}$ indicates a field transforming in the $(0, 1/2)$ representation. Note that $\psi^\dagger_\alpha = \bar{\psi}_{\dot{\alpha}}$.

(i) Correcting any misplaced signs or factors of 2, show that the following transformations together with their complex conjugates define a symmetry of the Lagrangian:

$$\delta_\zeta \phi = \sqrt{2} \zeta \psi, \quad \delta_\zeta \psi = i \sqrt{2} \sigma^\mu \bar{\zeta} \partial_\mu \phi - \sqrt{2} \zeta \lambda^* (\phi^*)^2. \quad (2)$$

Here ζ_α is a Grassmann (fermionic) parameter as in the BRST case, except ζ carries a spinor index.

(ii) Let's carry out a similar renormalization procedure to the one we used in lecture to compute the Yang-Mills β -function. Expand the action around a classical background to quadratic order in quantum fluctuations of the fields. Write down an expression for the 1-loop correction to the $|\phi^4|$ coupling. What happens in this case?

2. Consider a real scalar field of mass m coupled to photons,

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + A \phi \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}. \quad (3)$$

Use this Lagrangian to compute the tree-level decay rate of the scalar field the 2 photons.

3. Fun with decays: do P&S 19.2.

4. Oh no! It's 2-loop time, do P&S 17.1.