Problem Set 5

Physics 484

Due February 27

1. Pol. 11.5

2. Pol. 11.9

3. Pol. 11.10 (a) and (b)

4. Pol. 11.12

5. Learning about orbifolds: orbifold target spaces are constructed by choosing a space $\mathcal{M}$, and then quotienting by a discrete group, G. We have already seen two examples of orbifolds. The first is a string propagating on a circle, S^1, which we can view as the quotient $\mathbb{R}/\mathbb{Z}$. What is new in this case is the existence of winding modes, which are stringy soliton configurations. The second example was the equivalence described in problem 10.15 from the last problem set.

In this problem, I want you to develop some of the basics of orbifolds. Orbifolds provided some of the simplest early examples of stringy resolutions of classical curvature singularities. For example, the space $\mathbb{R}^4/\mathbb{Z}_2$ is defined as follows: take (ϕ_1, ϕ_2) as complex coordinates for $\mathbb{R}^4$ and quotient by the identification,

$$(\phi_1, \phi_2) \rightarrow (-\phi_1, -\phi_2). \tag{1}$$

Note that the origin is a fixed point of this action, and from the perspective of general relativity, there is a curvature singularity at the origin. We will see that perturbative string theory resolves this singularity!

(i) Take as your CFT two complex free bosons $(\phi_1(z,\bar{z}), \phi_2(z,\bar{z}))$ as above. Let us construct the $\mathbb{Z}_2$ orbifold CFT in steps. If this were a point particle theory i.e., quantum mechanics, how would you construct the orbifolded Hilbert space?

(ii) If you try to construct the orbifolded 2-dimensional CFT in the same way, do you get a modular invariant theory?

(iii) What (if anything) must you include to have a modular invariant theory? Construct the complete Hilbert space, and show that the partition sum is modular invariant.

(iv) An operator with weight $(h, \bar{h})$ is relevant if $h + \bar{h} < 2$ and irrelevant if $h + \bar{h} > 2$. If $(h, \bar{h}) = (1, 1)$, the operator is marginal as described in lecture. If you add an (ir)relevant or marginal operator to the action, what is the mass dimension of its coupling constant? As you flow into the infrared by renormalization group flow, what happens to (ir)relevant or marginal couplings?

(v) Lastly, extend your above construction of the orbifold CFT to the $(2, 2)$ free theory, i.e., add the relevant fermions. Do you find any marginal operators in the spectrum?