

Minimal Supersymmetric Standard Model (MSSM).

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1 Introduction

Even though the Standard Model has had years of experimental success, it has been known for a long time that it is not the ultimate theory of everything. The first step towards trying to build a more complete theory is the minimal supersymmetric standard model (MSSM). In the next section, I will list several problems apparent in the SM which prohibits it from laying claim to being the ultimate theory. In section 3, I will try to provide some motivation as to why the next step should be a supersymmetric extension for the SM. Section 4 deals with the actual technicalities of defining and building the MSSM, and finally section 5 presents the symmetry breaking terms that must be present in the MSSM lagrangian as well as the hope in the near future for experimental evidence of SUSY. Section 6 concludes this discussion.

2 Why not the Standard Model?

Even though the Standard Model (SM) has enjoyed years of success, it has been known for a long time that it is not the ultimate theory. Some problems were obvious from the beginning and as our experimental knowledge has grown, others have come into sight.

Experimentally, there is the neutrino mass problem – the SM predicts neutrinos having zero mass, whereas currently we have strong experimental evidence for massive neutrinos.

Theoretically, most of the problems are aesthetic in nature, but some are more severe, namely the incorporation of gravity. Any SM field theory which tries to include gravity ends up being non-renormalizable. So we know that, even if the SM is perfect in all other respects, it is at most an effective theory, without predictive power below the Planck scale.

The SM has, I believe, 19 free parameters to be determined experimentally, including all the particle masses and couplings. Also parameter values vary greatly with $m_t \sim 175\text{GeV}$ and $m_e \sim 0.5\text{MeV}$. This implies that $\frac{m_t}{m_e} \sim 10^5$. This contradicts the idea of QFT as a low energy effective theory. Additionally, there is no explanation for the triplication of generations or the mixing structure in the CKM matrix. Even though the weak CP violation can be incorporated in the SM via the CKM matrix, its origin is a mystery.

If we assume SM to be a low energy effective theory valid up to some energy scale Λ , the question arises, what is Λ ? We have three choices:

- There is only SM + gravity, implying that $\Lambda \sim M_{\text{planck}} \sim 10^{19}$ GeV.

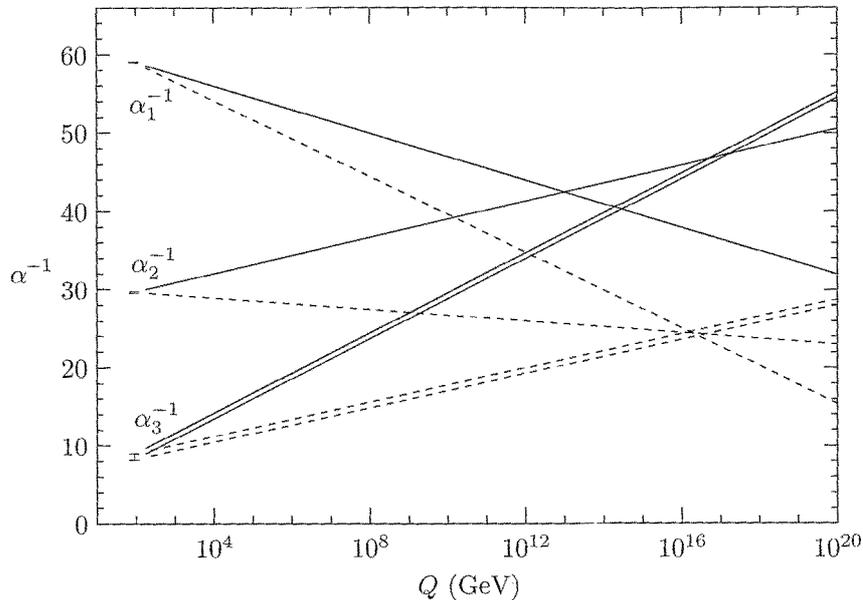


Figure 1: Solid lines denote SM couplings, dashed lines denote MSSM couplings.

- $SM \subset GUT + \text{gravity}$. $SU(3) \times SU(2) \times U(1) \subset SU(5)$. This implies that $\Lambda \sim \Lambda_{GUT} \sim 10^{16} \text{GeV}$.
- Something else interesting occurs at $\Lambda \ll \Lambda_{GUT}, M_{\text{planck}}$.

The first two choices don't really make much sense. To understand this consider the mass terms for the Higgs boson $m_h^2 \phi^* \phi$. As we shall see, there is no mechanism protecting the mass of the Higgs, so we would expect m_h to be of the order of the cut-off $\Lambda_{GUT}, M_{\text{planck}}$, but the experimental upper limit on m_h is about 100GeV [1]; this is known as the *hierarchy problem*. Also, if we look at the running of the couplings, within the framework of GUT, the result of the extrapolation is given in Fig.1 [1]. We see that that the coupling constants do come close together at very high energies, but do not actually meet; this is known as the *unification problem*. The dashed lines show the evolution of a modified set of renormalization group equations (SUSY), to be explained later, and these do meet accurately. All of these things are left unexplained by the first two hypotheses, so we hope for new physics at energy scales $\Lambda \sim 1-10 \text{TeV}$.

3 Why Supersymmetry?

Quadratic Divergences

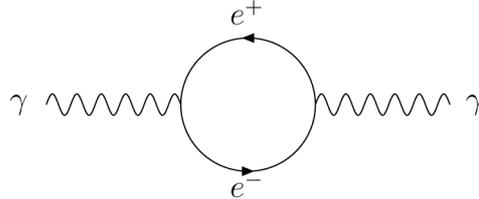


Figure 2: The photon self-energy in QED

I want to take a brief look at one loop corrections in QED, the best understood part of the SM [2]. My goal is to argue a connection between divergences and symmetries. First, investigating the photon propagator getting corrections from an electron loop as in Fig.2:

$$\begin{aligned}
 \pi_{\gamma\gamma}^{\mu\nu}(0) &= - \int \frac{d^4k}{(2\pi)^4} \text{tr} [(-ie\gamma^\mu) \frac{i}{\not{k} - m_e} (-ie\gamma^\nu) \frac{i}{\not{k} - m_e}] \\
 &= -4e^2 \int \frac{d^4k}{(2\pi)^4} \frac{2k^\mu k^\nu - g^{\mu\nu}(k^2 - m_e^2)}{(k^2 - m_e^2)^2} \\
 &= 0.
 \end{aligned} \tag{1}$$

This result is a consequence of the fact that the photon remains massless at all orders of perturbation theory as a result of Ward identities. Next if we consider the electron self energy of Fig.3:

$$\pi_{ee}(0) = \int \frac{d^4k}{(2\pi)^4} (-ie\gamma_\mu) \frac{i}{\not{k} - m_e} (-ie\gamma_\nu) \frac{-ig^{\mu\nu}}{k^2}$$

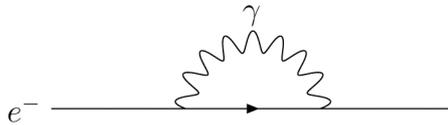


Figure 3: The electron self-energy in QED

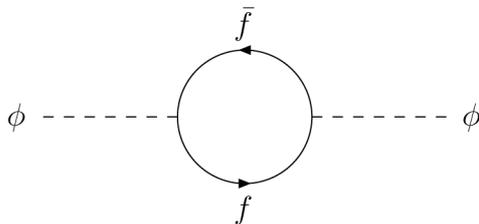


Figure 4: Fermion loop contribution to the self energy of the Higgs Boson

$$\begin{aligned}
&= -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_e^2)} \gamma_\mu (\not{k} + m_e) \gamma^\mu \\
&= -4e^2 m_e \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_e^2)}. \tag{2}
\end{aligned}$$

The integral in eq. 2 has a logarithmic divergence at large momenta. However the corrections to the electron mass are themselves proportional to the electron mass, and if we use the Planck scale as cut-off, we find a correction:

$$\delta m_e \approx 2 \frac{\alpha_{em}}{\pi} m_e \log \frac{M_{\text{planck}}}{m_e} \approx 0.24 m_e, \tag{3}$$

which is quite small. At a deeper level this small correction can be understood from a symmetry: in the limit $m_e \rightarrow 0$, the model becomes invariant under chiral rotation. If this symmetry were exact, the corrections in eq. 2 would have to vanish. In reality this symmetry is broken by the electron mass, so the correction must itself be proportional to m_e .

Now consider the massive fermion loop corrections to the propagator for the Higgs field, as shown in Fig.4. Let the $Hf\bar{f}$ coupling be given by λ_f ; the correction is then given by:

$$\begin{aligned}
\pi_{\phi\phi}^f(0) &= -N(f) \int \frac{d^4k}{(2\pi)^4} \text{tr} \left[\left(i \frac{\lambda_f}{\sqrt{2}} \right) \frac{i}{\not{k} - m_f} \left(i \frac{\lambda_f}{\sqrt{2}} \right) \frac{i}{\not{k} - m_f} \right] \\
&= -2N(f) \lambda_f^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + m_f^2}{(k^2 - m_f^2)^2} \\
&= -2N(f) \lambda_f^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right]. \tag{4}
\end{aligned}$$

Here, $N(f)$ is a multiplicity factor. The first term in the last line is quadratically divergent. If we were to replace the divergence Λ by M_{planck} , the resulting correction would be some thirty degrees of magnitude larger than the experimental bound on the Higgs. Also the correction itself is independent of m_h . This is related to the fact that setting $m_h = 0$ does not increase the symmetry group of the SM.

This divergence can, of course, be renormalized away in the usual way. However, for each order of perturbation theory, an extreme amount of fine tuning would be needed to cancel the divergences. Additionally that would still leave us large finite corrections at each loop order. Furthermore, there would be a contribution similar to eq. 4 due to any arbitrarily heavy particle that existed [3]. For example, suppose there exists a complex scalar particle S which couples to the Higgs via a term $-\lambda_s^2 H^2 S^2$, then the mass correction would be:

$$\Delta m_h = \frac{\lambda_s}{16\pi^2} [\Lambda^2 - 2m_s^2 \log(\frac{\Lambda}{m_s}) + \dots] \quad (5)$$

Even if one rejected the physical interpretation of Λ and used dimensional regulation instead, the term with m_s could not be eliminated without the unphysical tuning of a counter-term specifically for that purpose. So we see that the Higgs mass tends to be of the order of the *heaviest* mass of the theory. So, as was stated in the beginning, the Higgs mass is not protected in the way the photon and even the electron mass are protected, and should tend to be of the order of the energy cut-off, which makes it very difficult to understand why m_h is so small. This in effect is the *hierarchy* problem mentioned earlier.

Since we traced the divergence cancellations of the photon and electron to symmetries, we hope that some kind of symmetry will again come to the rescue. Looking closely at eq.4, 5 the bosonic and fermionic divergences have opposite signs, so we hope that some kind of symmetry relating fermions and bosons will work. Indeed the introduction of two complex fields with appropriately chosen masses and couplings does the trick. This brings us to what is known as supersymmetry (SUSY): each SM field gets a super partner with appropriate masses and couplings to insure the cancellation of these divergences to all orders.

4 MSSM

From now on, I will assume the reader is familiar with SUSY algebra and the formulation of superfields and superpotentials.

Chiral Supermultiplets

All of the SM fermions have the property that the left handed and right handed components transform differently under the gauge groups. Only chiral multiplets can contain fermions whose left-handed components transform differently from their right-handed partners under a gauge group [3]. Therefore in the MSSM, each of the fundamental particles must be in either a gauge or a chiral supermultiplet. The spin-0 superpartners of the SM fermions are appended with an *s*, for example, the electron superpartner is named *selectron*. Additionally the superpartners are denoted with a $\tilde{}$, for example, the left handed selectron is written \tilde{e}_L . Note that the subscript on the selectron refers to the handedness of the SM partner of the selectron since the selectrons are Spin-0. The left-handed and right-handed components of the fermions are separate two-component Weyl fermions with different gauge transformations. Therefore, they must have separate complex scalar partners. The gauge interactions of the scalar partners are the same as the corresponding SM fermion.

The Higgs boson must reside in a chiral multiplet since it is Spin-0. Actually it turns out that we need *two* multiplets. This can be simply understood from the fact that the superpotential is holomorphic, so the Higgs multiplet giving mass to the up-type quarks could not give mass to the down-type quarks. So to have masses for all the quarks, we need to have at least two Higgs fields, one with $Y = \frac{1}{2}$ and the other with $Y = -\frac{1}{2}$. Another reason for needing two multiplets is that, if we just had one, then the electroweak gauge symmetry would suffer a triangle gauge anomaly. This lists all the chiral supermultiplets needed for MSSM, and are summarized in Table 1.

It is interesting to note that the Higgs supermultiplet H_d , containing $H_d^0, H_d^-, \tilde{H}_d^0, \tilde{H}_d^0$ has exactly the same SM quantum numbers as left handed sleptons and leptons L_i , e.g. $(\tilde{\nu}, \tilde{e}_L, \nu, e_L)$ [2, 3]. We might therefore have put the Higgs and the electron in the same multiplet. This would imply that the Higgs boson and the sneutrino are the same particle. Even if we ignored the anomaly cancellation argument given above, unfortunately, this results in a neutrino whose mass would be in huge violation of experimental bounds. Thus all MSSM superpartners of the SM are indeed new particles.

Names		Spin 0	Spin $\frac{1}{2}$	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks	Q	$(\tilde{u}_L \tilde{d}_L)$	$(u_L d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
quarks	\bar{u}	\tilde{u}_R^*	u_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
($\times 3$ families)	\bar{d}	\tilde{d}_R^*	d_R^\dagger	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu} \tilde{e}_L)$	(νe_L)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
($\times 3$ families)	\bar{e}	\tilde{e}_R^*	e_R^\dagger	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	H_u	$(H_u^+ H_u^0)$	$(\tilde{H}_u^+ \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
	H_d	$(H_d^0 H_d^-)$	$(\tilde{H}_d^0 \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 1: Chiral supermultiplets of the MSSM

Gauge Supermultiplets

The vector bosons of the SM must clearly reside in gauge supermultiplets [3]. Their fermionic superpartners are referred to as *gauginos*. The color interaction of QCD is mediated via gluons, their superpartners are called *gluinos*. As before, a tilde is used to denote the superpartner. In the SM, after electroweak symmetry breaking the W^0, B^0 mix to give mass eigenstates Z^0 and γ . Similarly the gaugino mixture of \tilde{W}^0 and \tilde{B}^0 are called zino (\tilde{Z}^0) and photino ($\tilde{\gamma}$). If SUSY were broken, these would be mass eigenstates with masses m_Z and 0. Table 2 summarizes the gauge supermultiplets of MSSM.

Names	Spin $\frac{1}{2}$	Spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
Gluino, gluon	\tilde{g}	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\tilde{W}^\pm \tilde{W}^0$	$W^\pm W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	\tilde{B}^0	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

Table 2: Gauge supermultiplets in the MSSM

The chiral and gauge supermultiplets listed above make up the particle content of the MSSM. The most interesting feature of this theory is that none of the superpartners have yet been discovered experimentally. If SUSY were unbroken, there would exist the selectrons with $m_e = 0.511$ MeV. Similarly the massless gluinos and photinos must also exist. These particles should have been easily detected long ago. Therefore supersymmetry is a broken

symmetry. I will return to this topic later on, after having built the MSSM lagrangian.

Chiral Lagrangian

I am not going to use the superfield notation, simply because it is easier to compare the MSSM model with SM using the component fields of the superfields. The free part of the lagrangian is:

$$\mathcal{L}_{\text{free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i + W^{*i} W_i \quad (6)$$

with W as defined below. The most general renormalizable interactions for these fields can be written as:

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} W^{ij} \psi_i \psi_j - W^i W_i^* + c.c., \quad (7)$$

where W is the superpotential defined by:

$$\begin{aligned} W &= \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \\ W^i &= \frac{\partial}{\partial \phi_i} W, \\ W^{ij} &= \frac{\partial^2}{\partial \phi_i \partial \phi_j} W, \end{aligned} \quad (8)$$

So the complete chiral lagrangian density can be written as:

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}. \quad (9)$$

Gauge Lagrangian

The lagrangian density for a gauge supermultiplet is given by:

$$\mathcal{L}_{\text{gauge}} = -F_{\mu\nu}^a F^{\mu\nu a} - i\lambda^\dagger \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a \quad (10)$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c \quad (11)$$

is the usual Yang-Mills field strength, and

$$\begin{aligned} D_\mu \lambda^a &= \partial_\mu \lambda^a - gf^{abc} A_\mu^b \lambda^c, \\ D^a &= -g(\phi^* T^a \phi). \end{aligned} \quad (12)$$

The index a runs over the adjoint representation of the gauge group ($a = 1 \dots 8$ for $SU(3)_C$, $a = 1, 2, 3$ for $SU(2)_L$, $a = 1$ for $U(1)_Y$), and the T^a 's are the hermitian matrices of that representation.

SUSY Lagrangian for MSSM

Since supersymmetry commutes with gauge transformations, the scalar and the fermion fields must be in the same representation of the gauge group. To have a gauge invariant lagrangian, we need to turn the ordinary derivatives in eq.6 into covariant derivatives as defined above. This procedure accomplishes the purpose of coupling the gauge vector bosons to the scalars and fermions of the chiral multiplet. But, we also need to consider whether there are any other interactions allowed by gauge invariance involving just the vector multiplet gauge fields. In fact there are three terms that can be added which are renormalizable. Including these gauge interaction terms, the full lagrangian density for a renormalizable supersymmetric theory is:

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} - \sqrt{2}g[(\phi^* T^a \psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a \phi)] + g(\phi^* T^a \phi)D^a, \quad (13)$$

but with ordinary derivatives in $\mathcal{L}_{\text{chiral}}$ replaced by covariant derivatives. The superpotential for the MSSM is given by:

$$W_{\text{MSSM}} = \bar{u}\mathbf{y}_u QH_u - \bar{d}\mathbf{y}_d QH_d - \bar{e}\mathbf{y}_e LH_d + \mu H_u H_d. \quad (14)$$

The fields appearing in the superpotential are chiral superfields in Table 1. The dimensionless yukawa couplings are 3×3 matrices in family space. All the gauge and family indices in the above have been suppressed. The μ term is the SUSY version of the Higgs mass term in the SM. The feynman rules corresponding to all these interactions can be worked out simply in the usual manner. Note that now we have many additional interactions compared to the SM: the $\mathbf{y}_{u,d,e}$ do not simply imply Higgs-quark-quark and Higgs-lepton-lepton couplings, but also squark-Higgsino-quark and slepton-Higgsino-lepton interactions.

R-parity

The SM contains all the possible renormalizable interactions that are allowed by the $SU(3) \times SU(2) \times U(1)$ symmetries. The superpotential given in the previous section, on the other hand, is merely minimal, in the sense that it contains only terms which are needed to be consistent with SM

interactions. However, in principle we could add terms which are gauge invariant and analytic in the chiral superfields, but don't respect baryon number or lepton number. This would be disturbing since B and L violating processes have never been observed. The strongest constraint comes from experimental bounds on proton decay, which violates both B and L by 1 unit. These troubling terms can be eliminated by adding a new symmetry to the MSSM, called *R-parity* or equivalently *matter-parity*. These are conserved quantum numbers defined for each particle by:

$$\begin{aligned} P_M &= (-1)^{3(B-L)} \\ P_R &= (-1)^{3(B-L)+2s}, \end{aligned} \tag{15}$$

where s is the spin of the particle. The condition to be enforced is that any term in the lagrangian must have $P_M = 1$. It can be checked that all the good terms satisfy this condition and the troublesome ones are ruled out. Defining R-parity makes counting interactions very easy since all the SM particles have $P_R = 1$, while all the superpartners have $P_R = -1$. If R-parity is exactly conserved then there can be no mixing between sparticles and the SM particles. Additionally each interaction vertex in the theory must contain an even number of $P_R = -1$ sparticles. This gives us some extremely useful phenomenological results:

- The lightest sparticle, called the LSP must be absolutely stable. If it is electrically neutral then LSP would be an excellent candidate for dark matter.
- Each sparticle other than the LSP must decay into an odd number of LSPs.
- Sparticles can only be produced in even numbers from SM particles (e.g., in colliders).

Note that in the above, we *defined* MSSM to conserve R-parity. This seems a completely arbitrary process since the MSSM would not suffer any internal inconsistencies were we not to include it. However, several mechanisms in extensions of the MSSM have been proposed which do indeed impose this symmetry [3].

5 Soft Supersymmetry Breaking

As was mentioned earlier, if supersymmetry were an exact symmetry we would have been able to observe the selectrons, photinos and gluinos by now. Since we haven't, we know that this is a broken symmetry. From a theoretical perspective, it is expected that this symmetry is spontaneously broken (SB), analogous to the electroweak symmetry in the SM. Even though several models of SB symmetries have been proposed, as far as I can figure, there is no consensus of opinion about how it is broken. Nevertheless, certain assumptions can be made about the form of the SUSY breaking terms that appear in the lagrangian.

Returning to our original hierarchy problem, we were forced to introduce two complex scalar fields to cancel quadratic divergences. This also required that the couplings be related (e.g. $\lambda_s = |\lambda_f|^2$). Now, if the broken symmetry is to provide us with the solution to the hierarchy problem, then the same relationship between the couplings must be maintained. This in turn leads us to *soft* SUSY breaking terms, i.e. terms respecting the cancellation of the divergences.

Girardello and Grisaru [2] showed that to all orders of perturbation theory, inclusion of the following five terms still give the divergence cancellations:

- scalar mass terms $m_{\phi_i}^2 |\phi_i|^2$,
- trilinear scalar interactions $A_{ijk} \phi_i \phi_j \phi_k + h.c.$,
- gaugino mass terms $\frac{1}{2} m_l \bar{\lambda}_l \lambda_l$, where l labels group index,
- bilinear terms $B_{ij} \phi_i \phi_j + h.c.$, and
- linear terms $C_i \phi_i$.

Then the effective lagrangian for the MSSM can be written as:

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \quad (16)$$

where $\mathcal{L}_{\text{soft}}$ includes terms of the form listed above. If the largest mass scale associated with the soft terms is denoted by m_{soft} , then the corrections to the Higgs must be of the form:

$$\Delta m_h^2 = m_{\text{soft}}^2 \left[\frac{\lambda}{16\pi^2} \ln\left(\frac{\Lambda}{m_{\text{soft}}}\right) + \dots \right]. \quad (17)$$

Since the mass difference between known SM particles and their superpartners would be determined by the m_{soft} terms appearing in \mathcal{L}_{soft} , we know that the sparticle masses cannot be too large, else we would lose the cure found for the hierarchy problem. Therefore it is expected that the masses of the lightest few sparticles should be around 1 TeV or so.

6 Conclusion

I started off by listing a host of problems with the SM. In conclusion, now that MSSM and its additional symmetries have been defined, I take a brief look at the problems mentioned in the beginning with the SM and their possible solutions.

The hierarchy problem is solved by definition, since the superpartners to the SM fields were chosen to cancel all quadratic divergences, thus putting a control on the Higgs mass.

It also turns out that computing the running of the couplings in the MSSM shows that they do indeed unify at a scale $\lambda_{GUT} \sim 2 \times 10^{16}$ GeV (Fig.1). This apparent unification of the couplings could be accidental, but it also gives strong hope for some kind of a grand unified theory.

This still leaves us with the large number of free parameters and the incorporation of gravity. In fact, after including the SUSY breaking terms in the MSSM, we are left with something like a 100 free parameters to be determined experimentally. This seems to have intensified the problem rather than solving it. This is where it is instructive to introduce *supergravity*. This is based on the local version of SUSY. SUSY algebra shows that invariance under local SUSY transformations imply invariance under a local coordinate change. This is the basis of Einstein's theory of General Relativity. Therefore, local SUSY naturally includes supergravity. I will not go into the details here, but introducing supergravity implies that the SUSY breaking terms can be described using only three parameters [2].

Furthermore, SUSY comes out very naturally in superstring theory, which is currently our best hope for a theory of everything. Particularly, string theory automatically includes gravity as well as having no free parameters.

I would like to end with a few remarks about neutrino masses. It can be shown that any terms introducing neutrino masses are outside the SM. Again I will not go into details here, but some of these can be included in the MSSM. The interested reader is referred to [4] for a detailed discussion on several of these extensions.

References

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