Physics 234 Report: Gauge mediated supersymmetry breaking

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1 Introduction

From a phenomenological view, the motivation for supersymmetry(SUSY) is that it provides us with an elegant method of dealing with the hierarchy problem. Supersymmetry solves the hierarchy problem because $|\lambda_f|^2 = \lambda_s$, where λ_f and λ_s are higgs' coupling to the Standard Model(SM) fermions and their scalar superpartners(SP) respectively. Due to this relationship between the two coupling constants the Λ_{UV}^2 correction to the squared higgs mass due each of these particles cancel each other. Experimentally, however we see do not any of these super particles and therefore supersymmetry can only be an approximate symmetry of nature. So we would like to create a model in which supersymmetry is broken, but the corrections to squared higgs mass m_H^2 should still remain only logarithmic. $\mathcal{L}_{soft}$ is the part of the lagrangian that consists of terms that satisfy this requirement. The couplings need to have positive mass dimension because if m_{soft} is the largest mass scale of the soft terms then in the limit $m_{soft} \to 0$ supersymmetry has to be restored. So using dimensional analysis we see that soft terms cannot create Λ_{UV}^2 corrections to the higgs mass [?]. So in general we find that corrections to m_H^2 due to soft terms must be

$$\Delta m_H^2 \sim m_{soft}^2 \log(\Lambda_{UV}/m_{soft}) \tag{1}$$

The mass of the higgs is on the order of the electro weak scale so $m_H \sim 100$ GeV. Therefore $m_{soft} \sim$ TeV energy scale so that we do not spoil the solution to the hierarchy problem. Therefore the splitting in masses between the (SM) and (SP) particles should also be $\sim m_{soft}$. So using this approximate recipe we are able to generate realistic masses $m_Z, m_W = 80.4, 91.2$ [?] without too much fine-tuning.

So to develop realistic approximately supersymmetric models we need specify a mechanism by which the SUSY breaking terms can be generated. From analogy with electro weak breaking, we would like to break SUSY at tree level by giving a vacuum expectation value(VEV) to a chiral field. However, mass splittings generated by this approach cannot be $\sim$ TeV because of the supertrace theorem (assuming that there are only 3 generations of quarks). According to this theorem

$$\sum_J (-1)^{2J}(2J+1)M_J^2 = 0 \tag{2}$$

for any globally supersymmetric theory that is broken at tree level, where M_J is the tree level mass of a particle with spin J. If we consider the mass squared matrices for spin 0, 1 and 1/2 particles the tree level SUSY breaking terms only contribute to off diagonal terms except for the D-terms in the spin 0 mass squared matrix. Nevertheless as the D-term is proportional to t_A (the generators of the gauge group) the trace of this term disappears and so the weighted trace does not change under tree level SUSY breaking terms. Another, possibly more rigorous argument (independent of the number of quarks) by Dimopoulos and Georgi shows with tree level SUSY breaking terms create either an squark of charge $2e/3$ lighter than the u quark or an squark of charge $-e/3$ that is lighter than the d quark [?].

Now if SUSY cannot be broken spontaneously at tree level, it cannot be broken at any order in perturbation theory due to the non-renormalization theorem. But non-peturbative effects can still break SUSY and in analogy with chiral symmetry QCD we could have an asymptotically free gauge coupling that at a lower energy scale becomes strong and so produces a potential for scalar, whose VEV's then break supersymmetry. As there are no extra strong forces affecting observable quark and leptons (i.e. the "observable sector") SUSY breaking must occur in some "hidden sector", that does feel this new strong force. This breaking of supersymmetry has then to be communicated to the observable sector by the particles forming the "messenger sector". Gravitation and gauge fields are two most common ways in which SUSY breaking is believed to be communicated to the observable sector producing the soft breaking terms:

$$\begin{aligned}-\mathcal{L}_{soft} &= -\frac{1}{2}(m_{\lambda_g}\lambda_g^a\lambda_g^a + m_{\lambda_W}\lambda_W^i\lambda_W^i + m_{\lambda_B}\lambda_B\lambda_B + h.c.) + m_{\tilde{L}}^2\tilde{L}^\dagger\tilde{L} + \\ &\quad m_{\tilde{Q}}^2\tilde{Q}^\dagger\tilde{Q} + m_{\tilde{U}}2\tilde{U}^*\tilde{U} + m_{\tilde{D}}^2\tilde{D}^*\tilde{D} + m_{\tilde{E}}^2\tilde{E}^*\tilde{E} + m_1^2\Phi_1^\dagger\Phi_1 + \\ &\quad m_2^2\Phi_2^\dagger\Phi_2 - (B\mu\Phi_1^T i\tau_2\Phi_2 + h.c.) + (h_l A_l\Phi_1^\dagger\tilde{L}\tilde{E} + h_d A_d\Phi_1^\dagger\tilde{Q}\tilde{D} \\ &\quad -h_u A_u\Phi_2^T i\tau_2\tilde{Q}\tilde{U} + h.c.)\end{aligned} \tag{3}$$

where λ's are the gauginos, the $\tilde{Q}$ and $\tilde{L}$ are the left handed squark and slepton doublets, $\tilde{U}$, $\tilde{D}$ and $\tilde{E}$ are the corresponding singlets, Φ_1 and Φ_2 are the two higgs doublets and $i\tau_2$ is the completely antisymmetric second rank tensor. So for the gauge group to mediate SUSY breaking each of these soft terms must be produced by messenger sectors overlap with the hidden sector.

2 Overview of gauge mediated SUSY breaking

In this section we will study the method in which the soft terms acquire non-zero values due SUSY in the hidden sector[1].

[1]Most of this section has be taken from [?]

2.1 The minimal gauge mediated SUSY breaking model

The simplest model for gauge mediated SUSY breaking is called minimal model. In the minimal model we have a single chiral superfield S in the hidden sector that couples to chiral fields Φ_i and $\overline{\Phi}_i$ in the messenger sector so that the corresponding super potential term is

$$W = \lambda_{ij}\Phi_i S\overline{\Phi}_j. \tag{4}$$

SUSY is broken in the hidden sector so that S acquires a VEV in its scalar and auxiliary components to become

$$\langle S\rangle = M + \theta^2\langle F\rangle. \tag{5}$$

Therefore, the squared mass matrix of the scalar components of the messenger particles is then

$$\begin{pmatrix} (\lambda M)^\dagger(\lambda M) & (\lambda F)^\dagger \\ (\lambda F) & (\lambda M)(\lambda M)^\dagger \end{pmatrix} \tag{6}$$

in the basis $\begin{pmatrix} \Phi \\ \Phi^\dagger \end{pmatrix}$. As S is a singlet we can choose a gauge transformation that makes (λM) and (λF) both real and diagonal, so we can make with the redefinitions for $\lambda_{ii}M \to M_i$ and $\lambda_{ii}F \to F_i$. So squared masses of the messenger fields are $M_i^2 \pm F_i$) and $\sqrt{F}$ is the measure of SUSY breaking. These messenger particles must now couple to observable sector is such a way that the unification of the couplings is not disturbed. So we assume that they have the same ratios of the total traces of the squared $SU(3)\times SU(2)\times U(1)$ gauge generators as the quarks and leptons. We automatically satisfy this condition by requiring the messenger particles (together with some $SU(3)\times SU(2)\times U(1)$ neutral chiral fields) furnish a representation of some simple group $G \subseteq SU(3)\times SU(2)\times U(1)$, of which the quarks and leptons also form a representation. G is generally taken to be $SU(5)$ with messengers that transforms as a single flavor under the $\mathbf{5}+\overline{\mathbf{5}}$ representation (i.e. there are $SU(2)_L$ doublets l and $\bar{l}$ and $SU(3)_C$ triplets q and $\overline{q}$) .

Due to gauge invariance, the masses of vector boson and matter fermions are preserved. However, their superpartners have radiative corrections to their mass. So the mass and the couplings constants in eqn. **??** at an energy scale Q have following form:

$$t = \ln(M^2/Q^2) \tag{7}$$

$$\tilde{m}_{\lambda_a}(t) = k_r\frac{\alpha_r(t)}{4\pi}\Lambda_G \ (r=1,2,3), \tag{8}$$

$$m^2_{\tilde{f}}(t) = 2\sum_{r=1}^{3} C_r^{\tilde{f}} k_r \frac{\alpha^2(0)}{(4\pi)^2}\left[\Lambda_S^2 + h_r\Lambda_G^2\right] \tag{9}$$

$$m^2_{\tilde{Q}}(t) = 2\sum_{r=1}^{3}\frac{\alpha_r(0)^2}{(4\pi)^2}[(C_r^{\tilde{Q}} - \frac{K_t}{12}a_r)\Lambda_S^2 + h_r C_r^{\tilde{Q}}\Lambda_G^2]$$

$$
\begin{aligned}
& & -\frac{K_t}{6}(H_1 - K_1 H_2^2)\Lambda_G^2 & \quad (10)\\
m^2_{\tilde{t}_R}(t) &=& 2\sum_{r=1}^{3}\frac{\alpha_r(0)^2}{(4\pi)^2}[(C_r^{\tilde{t}_R} - \frac{K_t}{6}a_r)\Lambda_S^2 + h_r C_r^{\tilde{t}_R}\Lambda_G^2] & \\
& & -\frac{K_t}{3}(H_1 - K_1 H_2^2)\Lambda_G^2 & \quad (11)\\
A_t(t) &=& (H_3 - K_t H_2)\Lambda_G & \quad (12)\\
\mu(t) &=& \mu(0)(1-K_t)^{1/4}\Pi_{r=1}^{3}\left[\frac{\alpha_r(0)}{\alpha_r(t)}\right]^{\frac{a_r^{\mu}}{2b_r}} & \quad (13)\\
B(t) &=& B(0) + (H_4 - \frac{K_t}{2}H_2)\Lambda_G + \delta B^{(NLO)}(t) & \quad (14)\\
m^2_{\Phi_1}(t) &=& 2\sum_{r=1}^{3} C_r^{\Phi_1} k_r \frac{\alpha(0)^2}{(4\pi)^2}[\Lambda_S^2 + h_r\Lambda_G^2] + \delta m_{\Phi_1}^{(1-loop)}(t) & \quad (15)\\
m^2_{\Phi_2}(t) &=& 2\sum_{r=1}^{3} k_r \frac{\alpha(0)^2}{(4\pi)^2}[(C_r^{\Phi_2} - \frac{K_t}{4}a_r)\Lambda_S^2 + k_r h_r C_r^{\Phi_2}\Lambda_G^2] & \\
& & -\frac{K_t}{2}(H_1 - K_t H_2^2)\Lambda_G^2 + \delta m_{\Phi_2}^{(NLO)} + \delta m_{\Phi_2}^{(1-loop)} & \quad (16)
\end{aligned}
$$

where $k_1 = 5/3; k_2 = 1; k_3 = 1$ are chosen such that there is a unification at the GUT scale, n_i is the Dynkin index[2] of the i^{th} messenger, $\alpha_r(0)$ are the gauge couplings at the messenger scale M, $C_r^{\tilde{f}}$ is the quadratic casimir of the $\tilde{f}$ particle and b_r are the β-function coefficients

$$
b_3 = -3,\ b_2 = 1,\ b_1 = 11. \quad (17)
$$

The mass terms $m_{\tilde{f}}$ correspond superpartners of lower mass quarks and leptons, where the Yukawa terms have been neglected. However, for the stops whose squared mass matrix after electro weak breaking corresponds to

$$
\begin{pmatrix} m^2_{\tilde{Q}} + m_t^2 + (1/2 - 2/3\sin^2\theta_W)\cos 2\beta M_Z^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m^2_{\tilde{t}_R} + m_t^2 + 2/3\sin^2\theta_W\cos 2\beta M_Z^2 \end{pmatrix} \quad (18)
$$

the off diagonal terms are important so $m_{\tilde{Q}}$ and $m_{\tilde{t}_r}$ have to be modified. In eqn.**??** $\tan\beta = |v_2|/|v_1|$, θ_W is the Weinberg angle and M_Z is as usual the mass of the z-boson. The (NLO) terms correspond to terms that are suppressed by $1/t$ as compared to leading order and (1-loop) terms are contributions to the higgs masses due to the one loop effective potentials. The functions of t that appear in eqns.(8..16) are rather complicated can be referred to in the appendix. If as we assumed at the beginning of this section there is only a single S in the hidden sector that couples to the messenger sector then the gaugino and squark

[2] The Dynkin index is the ratio of the trace of squared casimir in the i^{th} representation to that in the adjoint representation and $N = \sum_i n_i$ is called the messenger index.

mass simplify to become

$$\Lambda_G = N\frac{F}{M}[1+\mathcal{O}(F^2/M^4)] \tag{19}$$

$$\Lambda_S = N\frac{F^2}{M^2}[1+\mathcal{O}(F^2/M^4)] \tag{20}$$

where $N = \sum_i n_i$ is called the messenger index and two loop corrections to the gaugino mass if the N and M are small.

From eqn. **??** and eqn. **??** for $F \ll M^2$ we find that the masses are independent of the Yukawa couplings between the messenger sector and hidden sector. Since gaugino masses arise at one-loop and scalar masses at two loops, super partner masses are generally the same order for particles with similar gauge charges. If the messenger scale M is much less than the GUT scale then we have $\alpha_3 \gg \alpha_2 > \alpha_1$ so the quarks and gluinos receive mass predominantly from $SU(3)_C$ interactions, the left-handed sleptons and W-inos from $SU(2)_L$ and the right-handed sleptons and B-ino from $U(1)_Y$ interactions. The masses of particles are therefore tightly correlated in the small F/M^2.

The parameter $(\alpha/4\pi)\Lambda$ sets the scale for soft mass which should be of order of the electro weak scale $\Lambda \sim \mathcal{O}(100\text{ TeV})$ so as not to restore hierarchy problem. Now as $\Lambda \sim F/M$ the messenger scale is arbitrary expect that $M > \Lambda$. Another property of the minimal messenger sector is that it is invariant under charge conjugation and parity. Therefore the $U_Y(1)$ Fayet-Iliopoulos D-term vanishes, which is important as this term induces soft scalar masses that have magnitudes much greater than the two-loop contributions and so would breaks $SU(3)_C$ and $U(1)_Q$. The $U(1)_Y$ D-term is only generated by gauge couplings to chiral standard model fields at the three loops.

The tri linear A-terms explicitly violate flavor symmetries, however as messenger is flavor blind in the observable sector these terms cannot be generated at one-loop. However, A-terms are generated in leading approximation by the RG evolution proportional to the gaugino masses and they vanish at the messenger scale. At low scales the A-terms have magnitudes $A \sim (\alpha/4\pi)m_\lambda \ln(M/m_\lambda)$, and so are small compared to other soft terms.

2.2 Variations on the minimal model

As should be clear from this discussion the minimal model puts very strong bounds on most of the supersymmetry breaking terms[3]. Therefore, we would like to know how these constraints change with variations in the minimal model. A possible generalization is to change the messenger sector representations such that they still are consistent with perturbative unification. In this case, the gaugino masses grow with the messenger index while the scalar masses grow like the square root of the messenger sector. Therefore, models with larger messenger sector representations have relatively heavier gauginos than the corresponding scalars as compared to the minimal model. Another possible generalization is

[3]Information for this section comes mainly from [**?**]

to introduce more fields in hidden sector that couple to the messenger sector. In this case, the messenger sectors' scalar and fermion masses are not in general aligned. Such a situation arises if the messengers receive masses from a sector not associated by SUSY breaking. The messengers can gain mass at the confinement or duality scale with SUSY broken at a lower scale. In these types of models, the scalar and gaugino masses are no long strongly constrained.

2.3 The μ problem

The most uncertain predictions of gauge mediated SUSY breaking has to do with μ. As is clear from eqn. **??**. The μ term has to already exist at the messenger scale and so cannot be completely generated radiatively in the gauge mediated mechanism. Also the μ cannot be set to zero as $B\mu$ and it are only terms in lagrangian that explicitly breaks the symmetry

$$\begin{aligned} \Phi_1 \rightarrow e^{i\phi}\Phi_1 \qquad & \Phi_2 \rightarrow e^{i\phi}\Phi_2 \\ Q \rightarrow e^{-i\phi}Q \qquad & V_i \rightarrow V_i \\ \overline{D} \rightarrow \overline{D} \qquad & \overline{U} \rightarrow \overline{U}. \end{aligned} \tag{21}$$

So the absence of μ would forbid the radiative generation of $B\mu$, but at electro weak scale $B\mu = m_{\tilde{f}}\ sin2\beta$. As the mass of scalars are not zero we have $\sin\beta = 0$ or in other words the either v_1 or v_2[4] are zero. This condition forces half the quarks to become mass less, which clearly is not commensurate with the standard model. So it is natural to generate the $B\mu$ term radiatively in which the symmetry under the transformation in **??** is broken by the term $\mu\Phi_1^T i\tau_2\Phi_2$. In addition, experimental bounds on chargino puts a puts the limit $\mu \sim 60$GeV. In this sense this term restores the hierarchy: instead of needing the higgs mass to be so much smaller than the Plank scale we now need μ to be much smaller than the Plank scale.

3 Conclusion

Theories involving gauge-mediated supersymmetry breaking provide an alternative to gravity mediated SUSY breaking, that solve the flavor-mixing problem. In addition, as the scale of the dynamics is low these models can be extremely predictive, with no a priori knowledge of quantum gravity. Gauge mediated theories have the advantage of being determined in terms of a few parameters: the most important of which is $\Lambda = F/M$ that sets scale at which supersymmetry is broken. The other important parameter is M whose lower bound determined from experimental limits on super partner masses and upper bound from requiring the gravity mediated contributions should be sufficiently small so as not to induce significant flavor mixing.

[4] v_i is the VEV of the i^{th} higgs field

Appendix

Here is a list of functions that appear in the masses and the Yukawa couplings of the SUSY breaking terms as listed in Section 2.

$$\Lambda_G = \sum_i n_i \frac{F_i}{M_i} g(F_i^2/M_i^4) \tag{22}$$

$$\Lambda_S = \sum_i n_i \frac{F_i}{M_i} f(F_i/M_i^2) \tag{23}$$

$$\alpha_r(t) = \alpha_r(0)\left[1+\frac{\alpha_r(0)}{4\pi}b_r t\right]^{-1} \tag{24}$$

$$\begin{aligned} g(x) &= \frac{1}{x^2}[(1+x^2)\ln(1+x)] + (x\to -x) \\ &= 1+\frac{x^2}{6}+\frac{x^4}{15}+\frac{x^6}{28}+\mathcal{O}(x^8) \end{aligned} \tag{25}$$

$$\begin{aligned} f(x) &= \frac{1+x^2}{x^2}[\ln(1+x) - 2Li_2(\frac{x}{1+x}) + \frac{1}{2}Li_2(\frac{2x}{1+x})] + (x\to -x) \\ &= 1+\frac{x^2}{36}-\frac{11}{450}x^4-\frac{319}{11760}x^6+\mathcal{O}(x^8) \end{aligned} \tag{26}$$

$$h_r = \frac{k_r}{b_r}\left[1-\frac{\alpha_r^2(t)}{\alpha_r^2(0)}\right] \tag{27}$$

$$a_r = 2(C_r^{\tilde{Q}} + C_r^{\tilde{t}_r} + C_r^{H_2}) = (13/9, 3, 16/3) \tag{28}$$

$$\begin{aligned} H_1 &= \frac{\alpha_X t_X}{4\pi} H_3\left(\frac{t}{t_X}-1\right) + \left(\frac{\alpha_X t_X}{4\pi}\right)^2 \left\{\left[\frac{E}{F}\left(\frac{t}{t_X}-1\right)+\frac{1}{F}\right]\right. \\ &\left. \sum_{r=1}^3 a_r\frac{\alpha_r(0)}{4\pi} + \sum_{r=1}^3 a_r b_r\frac{\alpha_r^2(0)}{(4\pi)^2} + \left[\sum_{r=1}^3 a_r\frac{\alpha_r(0)}{4\pi}\right]^2\right\} \end{aligned} \tag{29}$$

$$H_2 = \frac{\alpha_X t_X}{4\pi}\left[\frac{E}{F}\left(\frac{t}{t_X}-1\right)+\frac{1}{F}-\frac{1}{t_X}+\sum_{r=1}^3 a_r\frac{\alpha_r(0)}{4\pi}\right] \tag{30}$$

$$H_3 = \sum_{r=1}^3 \frac{a_r}{b_r}k_r\frac{\alpha_r(0)-\alpha_r(t)}{4\pi} \tag{31}$$

$$\alpha_X = \frac{k_s k_r - k_r b_s}{b_r\alpha_s^{-1}(0) - b_s\alpha_r^{-1}(0)} \quad \forall r\neq s \; r,s=1,2,3 \tag{32}$$

$$H_4 = \sum_{r=1}^3 \frac{a_r^\mu}{b_r}k_r\frac{\alpha_r(0)-\alpha_r(t)}{4\pi} \tag{33}$$

$$t_X = \frac{4\pi[k_r\alpha_s^{-1}(0) - k_s\alpha_r^{-1}(0)]}{k_s b_r - k_r b_s} \quad \forall r\neq s \; r,s=1,2,3 \tag{34}$$

$$E = \Pi_i^3\left[\frac{\alpha_r(0)}{\alpha_r(t)}\right]^{\frac{a_r}{b_r}} \tag{35}$$

$$F = \int_0^1 dtE \quad (36)$$

$$a_r^{\mu} = 2(C_r^{\Phi_1} + C_r^{\Phi_2}) = (1,3,0) \quad (37)$$

$$\delta B^{(NLO)}(t) = -\frac{\alpha_s^2(t)h_t^2(t)}{8\pi^4}\Lambda_G + \sum_{r=1}^{3} a_r^{\mu} k_r \frac{\alpha_r^2(t)}{(4\pi)^2}\Lambda_G \quad (38)$$

$$\delta m_{\Phi_2}^{(NLO)}(t) = -\frac{\alpha_s^2(t)h_t^2(t)}{8\pi^4}\Lambda_S, \quad (39)$$

where $h_t(t)$ is the running Yukawa coupling of the top to the higgs different is related to the top mass m_t and is not to be confused with h_r defined above. $K_t < 1$ as it is proportional to the square of h_t and the messenger scale is less than the GUT scale so h_t is small. The definitions of α_X and t_X are independent of the specific indices as by construction they are the coupling constants and mass scale respectively at unification.

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