Gauge-Mediated Supersymmetry Breaking

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The Standard Model has been a phenomenological success, for experiments at energy scales currently reachable have not confirmed any deviations from the theoretical predictions. However, the Standard Model is not at all complete, but a work in progress with additional structure being indicated by experiment. A unified theory would have to describe physics at arbitrary energy scales. Unfortunately, the Standard Model lacks that ability because of the infamous "hierarchy problem". The hierarchy problem lies in the dependence of the Higgs mass on the ultraviolet cut-off, Λ_{UV}, at which new physics exists. The Higgs potential is given by

$$V = m_H^2 |H|^2 + \lambda |H|^4$$

The Higgs acquires through this potential a non-zero expectation value. The mass of the Higgs is given by $m_H^2 = 2\lambda \langle H \rangle$. Experimentally (for example, from the decay rates of electrons and muons) it is known that $\langle H \rangle = 174 GeV$, which means that the mass of the Higgs should be of the order of 100 *GeV*. A one-loop correction to this mass, containing a fermion *f*, has ultraviolet divergences proportional to

$$\Delta m_H^2 \propto \left[-2\Lambda_{UV}^2 + 6m_f^2 \ln\left(\Lambda_{UV} / m_f \right) \right]$$

In the Standard Model, the biggest correction comes from the top quark. If Λ_{UV} is of the order of the reduced Planck scale $(8\pi G_{\text{Newton}})^{-1/2}$=2.4 10^{18}*GeV*, then the quantum correction is 30 orders of magnitude larger than the Higgs mass. Supersymmetry provides a solution to the hierarchy problem. The contributions to the Higgs mass from fermionic and bosonic loops have opposite signs and this leads to a systematic cancellation of the quadratic divergences. Therefore, one way of solving the hierarchy problem is to introduce an extended version of the Standard Model, in which every particle is given a superpartner. However, in reality we do not observe those superpartners, and therefore, supersymmetry must be broken.

Supersymmetry is broken if the expectation value of the scalar potential in the vacuum state is strictly positive. Equivalently, all F_i or D^a terms cannot simultaneously vanish for any values of the fields for supersymmetry to be spontaneously broken. The Fayet-Iliopoulos mechanism provides a non-zero D-term. It works if the gauge symmetry contains a *U(1)* factor. Unfortunately, this mechanism is difficult to implement in the MSSM. The $U(1)_Y$ symmetry cannot be used, because it leads to broken color and electromagnetism, but not supersymmetry. The other option is to assume the existence of some other *U(1)* symmetry, which has not been observed yet, because it is broken at some very high energy scale. In this case, it is very difficult to assign appropriate masses to all MSSM particles, especially the gauginos. More promising are models introduced by O'Raifeartaigh that depend on a non-zero F-term. Again, we would like to construct a superpotential *W* such that the equations $F_i = -\partial W^* / \partial \phi^{*i} = 0$ (where i runs through all superfields) do not have a simultaneous solution. It is clear that such a potential must contain a linear term, e.g.

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2$$

Because of the linear term, Φ_1 has to be a gauge-singlet. Unfortunately, none of the existing supermultiplets of the MSSM can fill this position, and therefore, the MSSM has to be extended. It is important to understand what effects break supersymmetry and how this affects the particles of the MSSM. It turns out that it is not possible to communicate the supersymmetry breaking to the MSSM particles by using renormalizable interactions at tree level. One reason is that supersymmetry does not allow interaction terms of the form (scalar)-(gaugino)-(gaugino), which are necessary for the gaugino to obtain mass at tree level. Another problem comes from the supertrace theorem. According to the theorem, at tree level, even after supersymmetry is broken, we have (the *M*'s are the corresponding mass matrices)

$$Tr\left(M^2_{real\ scalars}\right) = 2Tr\left(M^2_{chiral\ fermions}\right)$$

However, the masses of the chiral fermions are known to be very small, which means that some of the squarks or sleptons should have already been detected. For these reasons, the MSSM soft terms should arise radiatively, rather than from direct coupling to supersymmetry-breaking parameters. Thus, the supersymmetry breaking must occur in a "hidden sector" of particles, which are not directly coupled to the "observable sector" of our energy scale. However, there must be some mechanism that mediates supersymmetry breaking from the hidden sector to the observable sector. There are two main competing such mechisms—gravity mediated and gauge mediated. We will describe here the latter.

The basic idea of gauge-mediated supersymmetry breaking is to introduce some new superfields, called messengers, which couple directly to the supersymmetry-breaking parameters in the hidden sector and interact with the particles of the MSSM through the $SU(3)\times SU(2)\times U(1)$ gauge bosons and gauginos.

In the simplest model, the messenger fields are a set of supermultiplets $q, \bar{q}, l, \bar{l}$, which transform under the SM symmetry $SU(3)\times SU(2)\times U(1)$ as

$$q \sim \left(3,1,-\tfrac{1}{3}\right), \bar{q} \sim \left(\bar{3},1,\tfrac{1}{3}\right), l \sim \left(1,2,\tfrac{1}{2}\right), \bar{l} \sim \left(1,2,-\tfrac{1}{2}\right)$$

This is usually implemented by embedding the gauge group into a larger group that can contain the $r+\bar{r}$ representations of $SU(3)\times SU(2)\times U(1)$, for example *SU(5)*. These scalars together with their supersymmetric partners, the messenger quarks $\psi_q, \psi_{\bar{q}}$ and leptons $\psi_l, \psi_{\bar{l}}$, must have masses beyond the observable sector, otherwise they would have been observed by now. They acquire masses by coupling to a very heavy particle—a gauge singlet *S* from the hidden sector. The superpotential of the interaction is

$$W_{mess} = y_2 S l\bar{l} + y_3 S q\bar{q}$$

The scalar component of S and its auxiliary field F_S must both acquire vacuum expectation values, $\langle S\rangle$ and $\langle F_S\rangle$, through an O'Raifeartaigh model or some other mechanism. The exact process is not important for the moment and we can just assume that S participates in some other part of the superpotential, $W_{breaking}$. In the Lagrangian, the messenger quarks and leptons acquire mass terms

$$L = -y_2\langle S\rangle\psi_l\psi_{\bar{l}} - y_3\langle S\rangle\psi_q\psi_{\bar{q}} + c.c.$$

Neglecting the unimportant D-terms, the scalar potential for $q,\bar{q},l,\bar{l}$ becomes

$$V = \left|\frac{\partial W_{mess}}{\partial l}\right|^2 + \left|\frac{\partial W_{mess}}{\partial \bar{l}}\right|^2 + \left|\frac{\partial W_{mess}}{\partial q}\right|^2 + \left|\frac{\partial W_{mess}}{\partial \bar{q}}\right|^2 + \left|\frac{\partial W_{mess}}{\partial S} + \frac{\partial W_{breaking}}{\partial S}\right|^2$$

To finish the calculation, we need the reasonable assumption that

$$\left\langle\frac{\partial W_{breaking}}{\partial S}\right\rangle = -\langle F_S^*\rangle, \quad \left\langle\frac{\partial W_{mess}}{\partial S}\right\rangle = 0$$

Then, after replacing S and F_S with their vacuum expectation values, we obtain the following mass terms for the messenger scalar leptons and quarks

$$|y_2\langle S\rangle|^2\left(|l|^2 + |\bar{l}|^2\right) + |y_3\langle S\rangle|^2\left(|q|^2 + |\bar{q}|^2\right) - \left(y_2\langle F_S\rangle l\bar{l} + y_3\langle F_S\rangle q\bar{q} + c.c.\right)$$

We can read off the mass matrices and find the corresponding mass eigenvalues

$$l,\bar{l} : m^2_{fermions} = |y_2\langle S\rangle|^2, \quad m^2_{scalars} = |y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|$$
$$q,\bar{q} : m^2_{fermions} = |y_3\langle S\rangle|^2, \quad m^2_{scalars} = |y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|$$

If $\langle F_S\rangle$ is non-zero, there is a clear violation of supersymmetry. It is propagated to the MSSM gauginos through one-loops containing the messenger fields, Fig. 1. Computing these one-loop diagrams, one finds that the resulting MSSM gaugino masses are given by

$$M_a = \frac{k_a\alpha_a}{4\pi}\Lambda, \; a = 1,2,3$$

Here, the α's are the coupling constants of weak and strong interactions, normalized such that $k\alpha$'s are all equal at the GUT scale. Thus,

$$\alpha_1 = g'^2/4\pi, \; \alpha_2 = g^2/4\pi, \; \alpha_3 = g'^2\cos^2\theta_W/4\pi$$
$$k_1 = \frac{5}{3}, \; k_2 = k_3 = 1$$

The mass parameter Λ is defined as

$$\Lambda = \frac{\langle F_S\rangle}{\langle S\rangle}$$

If the expectation value of the auxiliary field were zero, then the messenger scalars would be degenerate with their superpartners and there would be no contribution to the gaugino masses. The corresponding gauge bosons cannot obtain masses through such processes, since they are protected by gauge invariance. Ward's identity guarantees that the contributions from all such diagrams will add up to zero.

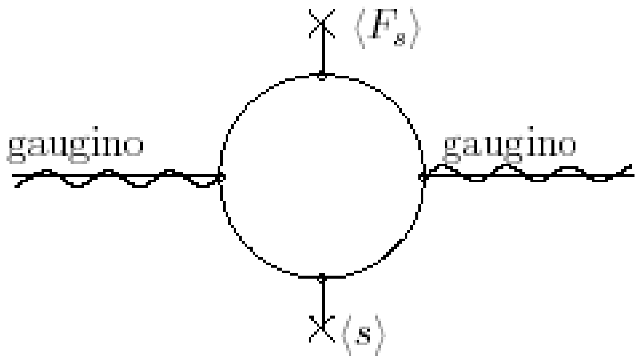

Fig.1: Radiative contributions to the MSSM gauginos masses (taken from Dubovsky, Gorbunov, Troitsky, hep-ph/9905466)

The equation for the gauginos' mass shifts holds at the heavy messenger mass scale. The running mass parameters can then be evolved down to the electroweak scale. The scalars of the MSSM do not get any mass corrections from one-loops. The leading contributions come from the two-loop diagrams shown in Fig. 2. The computation of these graphs gives the following mass shifts for each MSSM scalar:

$$m_\phi^2 = 2\Lambda^2\left[\left(\frac{k_3\alpha_3}{4\pi}\right)^2 C_3^\phi + \left(\frac{k_2\alpha_2}{4\pi}\right)^2 C_2^\phi + \left(\frac{k_1\alpha_1}{4\pi}\right)^2 C_1^\phi\right]$$

where the C_a^ϕ's are the Casimir operators for each MSSM scalar for each gauge group. Explicitly, we have

$$C_3^\phi = \begin{cases} 4/3 \text{ for } \phi = \tilde{Q}_i, \tilde{u}_{Ri}, \tilde{d}_{Ri} \\ 0 \text{ for } \phi = \tilde{L}_i, \tilde{e}_{Ri}, H_u, H_d \end{cases}$$

$$C_2^\phi = \begin{cases} 3/4 \text{ for } \phi = \tilde{Q}_i, \tilde{L}_i, H_u, H_d \\ 0 \text{ for } \phi = \tilde{u}_{Ri}, \tilde{d}_{Ri}, \tilde{e}_{Ri} \end{cases}$$

$$C_1^\phi = 3Y_\phi^2/5 \text{ for each } \phi \text{ with hypercharge } Y_\phi$$

Again, the mass shifts are evaluated at the scale of the messenger fields. At this scale, the Yukawa couplings can be neglected to a very good approximation. However, when the mass shift parameters are evolved down to the electroweak scale, there will be non-zero contributions from the Yukawa couplings. This effect will be largest for the third family of squarks and sleptons (the superpartners of the top and bottom quarks) and has to be taken into account in more complex models.

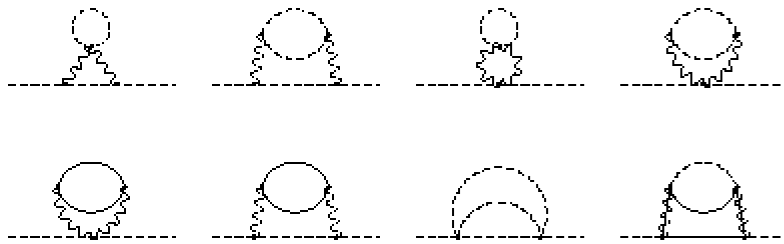

Fig. 2: Two-loop contributions to the MSSM scalar masses (taken from S. Martin, "A Supersymmetry Primer"). The messenger scalars and fermions correspond to the dashed and solid lines in the loops.

From the equations for the masses of the messenger fields, MSSM gauginos and scalars, it is clear that these corrections are quite small unless there are very large hierarchies in the messenger sector, meaning that the ratio $\langle F_S\rangle / \langle S\rangle$ is large.

The model that was just described is called the minimal model of gauge-mediated supersymmetry breaking. Let us now consider a more complicated messenger sector, consisting of N_f flavors of superfields $\Phi_i, \overline{\Phi}_i, i = 1,.., N_f$ transforming as the $r + \overline{r}$ representation of the gauge group. The interaction superpotential term is given in analogy with the minimal model:

$$W = \lambda_{ij} \overline{\Phi}_i S \Phi_j$$

After replacing S with its expectation value, we find the mass matrix for the fermionic components of the messenger fields to be $\lambda\langle S\rangle$. The messenger scalars obtain a squared-mass matrix

$$\begin{pmatrix} \Phi^+ & \overline{\Phi} \end{pmatrix} \begin{pmatrix} (\lambda\langle S\rangle)^+ (\lambda\langle S\rangle) & \lambda\langle F\rangle \\ \lambda\langle F\rangle & (\lambda\langle S\rangle)(\lambda\langle S\rangle)^+ \end{pmatrix} \begin{pmatrix} \Phi \\ \overline{\Phi}^+ \end{pmatrix}$$

We can diagonalize λ, and in this basis the mass^2-eigenvectors and values are given by $\left(\Phi_i + \overline{\Phi}_i^+\right)/\sqrt{2}$, $\left(\overline{\Phi}_i - \Phi_i^+\right)/\sqrt{2}$ and $M_i^2 \pm F_i$ respectively, where $M_i = \lambda_{ii}\langle S\rangle$, $F_i = \lambda_{ii}\langle F\rangle$. These equations are in complete analogy with the messenger mass equations in the minimal model.

The one- and two-loop calculations that give masses to the gauginos and the sparticles are evaluated at the messenger mass scale, since they involve loops with messenger particles in them. However, we would like to know exactly how much supersymmetry breaking occurs in our observable sector. This requires solving the renormalization group differential equations and evolving the coupling constants of the theory. An important parameter that arises is $t = \ln\left(\langle S\rangle^2 / Q^2\right)$, where Q is our low-energy scale. Having this in mind, we can calculate the loops in Fig.1.

Let's start with the assumption that $\langle F\rangle << \langle S\rangle^2$. This approximation allows one to find the mass shifts without directly calculating the corresponding Feynman diagrams (see [2]). The gaugino masses are

$$M_a = \frac{k_a \alpha_a(t)}{4\pi}\Lambda_G \ , \quad a = 1,2,3$$

The $k\alpha$'s have the same normalization condition as for the minimal model. The last factor in the equation is defined as

$$\Lambda_G = \sum_{i=1}^{N_f} n_i \frac{F_i}{M_i}\left[1 + O\left(F_i^2 / M_i^4\right)\right]$$

with n_i being the Dynkin index of the gauge representation $r + \bar{r}$ with flavor index i. The energy scale dependence of the α's is given by

$$\alpha_r(t) = \alpha_r(0)\left[1 + \frac{\alpha_r(0)}{4\pi} b_r t\right]^{-1}$$

Here, the b's are the β-function parameters, b_3=-3, b_2=1 and b_1=11, and $\alpha_r(0)$ are the gauge coupling constants at the messenger mass scale.

Neglecting Yukawa coupling effects, the MSSM scalar masses at the scale Q are given by

$$m_\phi^2(t) = 2\sum_{r=1}^{3} C_r^\phi k_r \frac{\alpha_r^2(0)}{(4\pi)^2}\left[\Lambda_S^2 + h_r \Lambda_G^2\right],$$

$$h_r = \frac{k_r}{b_r}\left[1 - \frac{\alpha_r^2(t)}{\alpha_r^2(0)}\right]$$

Here, the C_r^ϕ's are the usual quadratic Casimir operators and Λ_S^2 is defined as

$$\Lambda_S^2 = \left(\sum_i^{N_f} n_i\right)\frac{\langle F\rangle^2}{\langle S\rangle^2}\left[1 + O\left(\langle F\rangle^2 / \langle S\rangle^4\right)\right]$$

The calculations above required the approximation $\langle F\rangle << \langle S\rangle^2$ and do not work in the case $\langle F\rangle \approx \langle S\rangle^2$. Explicit Feynman diagram calculations are necessary. It can be shown (see for example Dimopoulos, Guidice, Pomarol, , Phys. Lett. B 389, 37 (1996)) that the needed modifications to the equations above are

$$\Lambda_G = \sum_{i=1}^{N_f} n_i \frac{F_i}{M_i} g\left(F_i / M_i^2\right)$$

$$g(x) = \frac{1}{x^2}\left[(1+x)\ln(1+x)\right] + (x \to -x)$$

And

$$\Lambda_S^2 = \sum_{i=1}^{N_f} n_i \frac{F_i^2}{M_i^2} f\left(F_i / M_i^2\right)$$

$$f(x) = \frac{1+x}{x^2}\left[\ln(1+x) - 2Li_2\left(\frac{x}{1+x}\right) + \frac{1}{2} Li_2\left(\frac{2x}{1+x}\right)\right] + (x \to -x)$$

It is interesting to mention at this point, that the leading contributions to $\Lambda_{G,S}$ are independent of the coupling constants and depend only on the ratio $\langle F \rangle / \langle S \rangle^2$. However, this quantity is different for messengers with different SM quantum numbers. Thus, the inclusion of the correction functions *f* and *g* can be relevant only for some messengers. This leads to uncertainties in the mass prediction, which is typically small for squarks and sleptons, but could be up to 40% for gauginos (see [2]).

The scalar masses computations shown above neglected Yukawa-coupling effects. As was mentioned in the minimal model, these effects are greatest for the third generation sparticles. To describe these effects, we need the following matrix:

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{Q}_L}^2 + m_t^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\cos 2\beta M_Z^2 & m_t\left(A_t - \mu\cot\beta\right) \\ m_t\left(A_t - \mu\cot\beta\right) & m_{\tilde{t}_R}^2 + m_t^2 \frac{2}{3}\sin^2\theta_W \cos 2\beta M_Z^2 \end{pmatrix}$$

(μ and β are parameters put in by hand and responsible for the electroweak symmetry breaking; μ is the Higgs mixing mass, defined by the superpotential $\mu H_u H_d$ and its presence in the matrix is a main problem in theories with gauge mediation; for more information see [2])
The corresponding third-generation supersymmetry masses at the scale Q are given by

$$m_{\tilde{Q}_L}^2(t) = 2\sum_{r=1}^{3} k_r \frac{\alpha_r^2(0)}{(4\pi)^2}\left[\left(C_r^{\tilde{Q}_L} - \frac{K_t}{12} a_r\right)\Lambda_S^2 + h_r C_r^{\tilde{Q}_L}\Lambda_G^2\right]$$
$$- \frac{K_t}{6}\left(H_1 - K_t H_2^2\right)\Lambda_G^2$$

And

$$m^2_{\tilde{t}_R}(t) = 2\sum_{r=1}^{3} k_r \frac{\alpha_r^2(0)}{(4\pi)^2}\left[\left(C_r^{\tilde{t}_R} - \frac{K_t}{6} a_r\right)\Lambda_S^2 + h_r C_r^{\tilde{t}_R}\Lambda_G^2\right]$$
$$- \frac{K_t}{3}\left(H_1 - K_t H_2^2\right)\Lambda_G^2$$

Where $a_r = 2\left(C_r^{\tilde{Q}_L} + C_r^{\tilde{t}_R} + C_r^{H_2}\right) = (13/9, 3, 16/3)$ and the other parameters are defined below

$$E = \prod_{r=1}^{3}\left[\frac{\alpha_r(0)}{\alpha_r(t)}\right]^{\frac{a_r}{b_r}}, \quad F = \int_0^t dt\, E$$

$$H_1 = \frac{\alpha_X t_X}{4\pi} H_3 \frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \left(\frac{\alpha_X t_X}{4\pi}\right)^2 \left\{\left[\frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \frac{1}{F}\right]\sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi}\right\}$$
$$+ \left(\frac{\alpha_X t_X}{4\pi}\right)^2 \left\{\sum_{r=1}^{3} a_r b_r \frac{\alpha_r^2(0)}{(4\pi)^2} + \left[\sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi}\right]^2\right\}$$
$$H_2 = \frac{\alpha_X t_X}{4\pi}\left[\frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \frac{1}{F} - \frac{1}{t_X} + \sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi}\right]$$
$$H_3 = \sum_{r=1}^{3} \frac{a_r}{b_r} k_r \frac{\alpha_r(0) - \alpha_r(t)}{4\pi}$$

The α_X and t_X are defined by

$$\alpha_X = \frac{k_s b_r - k_r b_s}{b_r \alpha_s^{-1}(0) - b_s \alpha_r^{-1}(0)}, \forall r \neq s, \quad r, s = 1,2,3$$
$$t_X = \frac{4\pi\left[k_r \alpha_s^{-1}(0) - k_s \alpha_r^{-1}(0)\right]}{k_s b_r - k_r b_s}, \forall r \neq s, \quad r, s = 1,2,3$$

The definitions of α_X and t_X are independent of the specific indices *r* and *s*, because we are assuming gauge-coupling unification.
At the end,

$$K_t = \frac{6F}{E} \frac{h_t^2(t)}{(4\pi)^2}$$

Here $h_t(t) = \left(2\sqrt{2}G_F\right)^{1/2} m_t / \sin\beta$ is the running top quark Yukawa coupling.

One of the main advantages of gauge-mediated supersymmetry breaking theories over theories in which gravity is employed is that supersymmetry breaking is communicated to the observable sector through flavor blind interactions. The MSSM soft terms are generated at low mass scales and are not sensitive to the high-energy sector, where it is assumed flavor symmetry is broken. Also, gauge-mediated models can be completely described in the framework of field theory without involving quantum gravity effects or string theory. Such models have very few free parameters and theoretical predictions do not contradict experimental observations.

However, such theories also have their drawbacks. One problem is in the choice of group that contains the gauge group. In the case of *SU(5)*, messenger particle with masses much below the GUT scale will affect the running of the gauge couplings and might destroy the expected unification of the constants. Another problem is obtaining a correct pattern of the electroweak symmetry breaking. It requires the introduction of new parameters. Such a problem is the origin of the Higgs mixing mass, although certain progress has been maid towards a more direct generation of μ from the sector, which breaks supersymmetry.

Reference:

[1] S. Martin, A Supersymmetry Primer, hep-ph/9709356

[2] G.F. Guidice, R. Rattazzi, Theories with Gauge-Mediated Supersymmtry Breaking, hep-ph/9801271

[3] S. Dubovsky, D. Gorbunov, S. Troitsky, Gauge mechanism of mediation of supersymmetry breaking, hep-ph/9905466