Topological Defects in SUSY

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1 Introduction

Topological defects are general consequences of almost all symmetry breaking processes. They arise naturally when a system goes through a phase transition from a symmetric phase to a phase in which the symmetry is broken. The formation and the evolution of cosmic topological defects in the early (or late) Universe has many direct and observable consequences that may allow us to constrain possible models of physics at high energy scales.

In this paper, we discuss the formation, stability, and evolution of topological defects arising from standard field theory and in supersymmetric field theories. Then, we review how cosmological observations constrain the various theories.

2 What are topological defects?

Topological defects, in general, are regions of one phase embedded in a region of another, distinct phase. There are four different types of topological defects: domain walls, strings, monopoles, and textures. Domain walls are two dimensional, sheet like regions, strings are one dimensional, string like regions, and monopoles are point like "regions" of one phase in another phase. Textures are delocalized region of continuous phase transition that arise when large symmetry groups are completely broken. Many examples of topological defects are known to exist and have been studied in condensed matter laboratories. For example, magnetic domain walls of magnetized material and string defects in liquid Helium superfluid phase transitions have been extensively studied.

Of interest to us is the realization that topological defects similar to the ones studies in condensed matter arise naturally during the phase transitions associated with symmetry breaking of high energy physics models. Various defects will form during the electroweak phase transition, Grand Unified Theory (GUT) phase transition, and any super symmetry (SUSY) breaking that must happen if SUSY were to be real.

3 Condition for their existence

The existence of the various types of topological defects is model depended. The symmetry group structure of the unbroken phase and the broken phase of a theory determines the vacuum manifold, ie, the space of all possible ground states. Mathematically speaking, if the original symmetry group is denoted by $\mathbf{G}$ and the symmetry group of the broken phase is denoted $\mathbf{H}$, the vacuum manifold is the coset space of $\mathbf{G}$ and $\mathbf{H}$ denoted by $\mathcal{M} = \mathbf{G}/\mathbf{H}$.

The topology of the vacuum manifold determines which types of topological defects are allowed. The concept of homotopy groups allows us to concretely relate the existence of topological defects to the vacuum manifold structure. Homotopy group is a group of equivalent classes of homotopic loops, loops that can be smoothly deformed into each other without leaving the given manifold. They are denoted as $\pi_n(\mathcal{M})$, where n refers to the dimensionality of the loops in question. For example, the homotopy group $\pi_1(\mathcal{M})$ is the group consisting of homotopy classes of 1D loops. Any loop in a given homotopy class can be smoothly deformed into any other loop in the class, but cannot be deformed into loops in other classes. If $\mathcal{M}$ is the surface of a 1-sphere, ie a circle, any loop that winds around the circle once can be deformed into each other, but it cannot be deformed into a loop that winds around the circle once. On the surface of a 2-sphere, any loop is deformable to any other loop and hence there is only 1 element in the homotopy group, namely the identity element.

The rule of thumb is that if the homotopy group of the is trivial, then there cannot be any topological defects corresponding to that homotopy group of the theory. So, if $\pi_0(\mathcal{M}) = \mathbf{1}$, then the theory does not allow domain walls to form, although $\pi_0(\mathcal{M}) \neq \mathbf{1}$ is not a sufficient condition for their existence. The group $\pi_1(\mathcal{M})$ corresponds to cosmic strings, and the group $\pi_2(\mathcal{M})$ corresponds to monopoles. Note that if $\pi_0(\mathcal{M}) \neq \mathbf{1}$ for a given theory, then automatically $\pi_n(\mathcal{M}) \neq \mathbf{1}$ for any n greater than 0 for the same theory. Thus, if there is a chance that domain walls can form in one theory, then there's also a chance that cosmic strings and monopoles can form. This leads to the generality of monopole formation; in almost any theory, monopoles are formed.

4 So, how do Topological Defects Form?

First, for topological defects to form, there must be some symmetry breaking process. For concreteness, let's consider the simplest case: a complex scalar field $\phi(x)$ described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi + \frac{1}{2}m^2|\phi|^2 - \frac{\lambda}{4!}|\phi|^4, \tag{1}$$

where λ and m are some real constants. This Lagrangian exhibits a global U(1) symmetry under the transformation $\phi \to \phi e^{i\alpha}$ for some constant α. The vacuum solution to this model has a non-zero expectation value given by $\langle|\phi_0|\rangle^2 = 6m^2/\lambda$ and is not invariant under the U(1) symmetry transformation since the solution can have any complex phase.

At finite temperatures, the quantity to minimize becomes the free energy of the system. In effect, this modifies the zero temperature potential to an effective potential given by

$$V_T(\phi) = -\frac{1}{2}m^2(T)\phi^2 + \frac{\lambda}{4!}\phi^4 \tag{2}$$

where the mass $m^2(T) = m^2(1 - T^2/T_c^2)$ is now temperature dependent and T_c is the critical temperature above which the symmetry is restored.

4.1 Kibble Mechanism

The method of choice to form topological defects in current literature is the Kibble mechanism.[3] In this scenario, the process of defect formation begins with the Universe in the original unbroken phase. When the temperature of the Universe becomes lower than the critical phase transition temperature T_c, the symmetry breaking does not happen uniformly due to small quantum fluctuations.

Instead, small regions in the Universe begins the transition first, forming domains of broken phase (proto-domains) in a sea of unbroken phase. The sizes of these domains are determined by the characteristic length of correlations of ϕ^4 potential, ie, $\xi = m(T)^{-1} \simeq \sqrt{\lambda}|\langle\phi\rangle|$. Because domains separated by more than the correlation length are not in causal contact, these domains will in general break into different phase of the vacua. As the Universe cools further, these proto-domains expand in size and eventually come in contact with the other domains, forming discontinuity in field configuration at the interfaces of the domains.

For reasons that will become clear later in the paper, both monopoles and domain walls are completely ruled out in the Universe today. For this reason, we concentrate on the formation and properties of cosmic string solutions.

4.2 Nielson-Olesen Strings

4.2.1 Global strings

Let's consider the simplest theory which exhibits string solutions.[5] We start with the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) \tag{3}$$

where

$$V(\phi) = \frac{1}{2}\lambda \left(|\phi|^2 - \frac{1}{2}\eta^2\right)^2, \tag{4}$$

and λ and η are real constants. The Lagrangian is invariant under a global U(1) transformation of the form $\phi \to \phi e^{i\alpha}$ with α constant. We can solve for the ground state by solving $\partial V(\phi)/\partial\phi = 0$, which gives us the vacuum solution $\phi_0 = (\eta/sqrt2)\exp(i\alpha_0)$ with some constant α_0. The state is obviously not invariant under the symmetry transformation (ie, $\phi_0 \neq \phi_0 \exp(i\alpha)$ for any α), and therefore the symmetry is broken by the vacuum. The equation of motion corresponding to the Lagrangian can be obtained from the Euler equation and is given by

$$(\partial^2 + \lambda(|\phi|^2 - \frac{1}{2}\eta^2))\phi = 0. \tag{5}$$

Now, we look for a string solution, ie, a static, cylindrically symmetric solution with non-zero energy density. We make an ansatz of the form

$$\phi = \frac{\eta}{2} f(m_s \rho) e^{in\varphi} \tag{6}$$

where ρ, φ, z are the cylindrical coordinates, n is an integer, and m_s is the mass of the scalar field in the symmetry breaking vacuum. Substituting the ansatz into the equation of motion, we obtain an ordinary differential equation for f,

$$\frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\xi}\frac{\partial f}{\partial \rho} - \frac{n^2}{\xi^2} f - \frac{1}{2}(f^2 - 1) f = 0, \tag{7}$$

where $\xi \equiv m_s \rho$. The boundary condition at $\rho = 0$ is determined by requiring that ϕ be continuous, while the boundary condition at $\rho = \infty$ is determined by requiring that ϕ approach the vacuum solution. These conditions require that $f \to 0$ as $\rho \to 0$ and $f \to 1$ as $\rho \to \infty$. The solution describes a field configuration which is invariant under the symmetry transformation near the origin (ie, $\phi = 0$ at $\rho = 0$) and is in the symmetry breaking vacuum at far away. Since the solution is cylindrically symmetric, it describes a string of unbroken phase located at $\rho = 0$ embedded in broken phase.

4.2.2 Local strings

String solutions also exist when the internal symmetry is a local symmetry, in which a gauge field is introduced. The Lagrangian density for the model (Abelian Higgs model) becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu nu} + |D_\mu\phi|^2 - V(\phi), \tag{8}$$

where, as usual, $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The U(1) symmetry transformations are given by

$$\phi \to \phi e^{i\Lambda(x)} \tag{9}$$

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \Lambda(x), \tag{10}$$

where Λ is a real function. If we choose the radial gauge ($A_\rho = 0$), we make cylindrical symmetric ansatz analogously to the case of global U(1) symmetry,

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s\rho)e^{in\varphi} \tag{11}$$

$$A_i = \frac{n}{e\rho}\varphi_i a(m_s\rho), \tag{12}$$

where we have 2 form functions, $f(\xi)$ and $a(\xi)$. Substituting the form functions into the equation of motion, we once again obtain solutions in which the gauge symmetry is unbroken near the origin and broken outside.

One difference between a global string and a local string is that, as $\xi \to \infty$, form function for global strings approach 1 like ξ^{-2} whereas those of local strings approach 1 like $\exp(-\xi)$. This translates to the fact that the energy densities at large distances fall off faster for local strings. In fact, for global strings, the energy fall off is slow enough that the total energy of the string becomes infinite, while we don't have such problems in local strings.

Another difference is that local strings carry quantized magnetic flux given by

$$\int d^2x \vec{B} \cdot \hat{z} = \int_{S^1_\infty} dx^i A^i = \frac{2\pi n}{e}, \tag{13}$$

where S^1_∞ denotes a circle of infinite radius centered on the string.

4.3 SUSY Strings

There are also string solutions in the SUSY breaking phase transition. We consider the supersymmetric extension of the Abelian Higgs model given above.[1] The SUSY version consist of one vector potential $V(x, \theta, \bar{\theta})$ and m chiral superfields Φ_i, where $i = 1, ...m$. The most general Lagrangian density that is symmetric under local U(1) is given by

$$\mathcal{L} = \frac{1}{4}(W^\alpha W_\alpha|_{\theta\theta} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}) + (\bar{\Phi}_i e^{gq_i V}\Phi)|_{\theta\theta\bar{\theta}\bar{\theta}} + W(\Phi_i)|_{\theta\theta} + \bar{W}(\bar{\Phi}_i)|_{\bar{\theta}\bar{\theta}} + \kappa D, \tag{14}$$

where

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \tag{15}$$

D_α and $\bar{D}_{\dot{\alpha}}$ are the supersymmetric covariant derivatives, V is the vector superfield, W is the superpotential, and κD is the Fayet-Iliopoulos term which is invariant only under U(1) transformation. The most general renormalizable superpotential is

$$W(\Phi_i) = a_i\Phi_i + \frac{1}{2}b_{ij}\Phi_i\Phi_j + \frac{1}{3}c_{ijk}\Phi_i\Phi_j\Phi_k \tag{16}$$

which leads to a scalar potential of the form

$$U = |F_i|^2 + \frac{1}{2}D^2 = |a_i + b_{ij}\phi_j + c_{ijk}\phi_j\phi_k|^2 + \frac{1}{2}\left(\kappa + \frac{g}{2}q_i\bar{\phi}_i\phi_i\right)^2, \tag{17}$$

where F_i, D, and ϕ are the component fields of Φ and V.

4.3.1 F Strings

Now, we constrain the model further to consider the simplest model with vanishing Fayet-Iliopoulos term. This model is sometimes called F string model.

The simplest model that does not posses gauge anomaly consist of three chiral superfields with U(1) charges $0, 1$, and -1. For the sake of simplicity (and of finishing this paper on time), we only consider the bosonic fields The superpotential for the model is

$$W(\Phi_i) = \mu\Phi_0(\Phi_+\Phi_- - \eta^2) \tag{18}$$

for some real constant μ and η. The minimum of the scalar potential occurs when $\phi_0 = 0, \phi_+\phi_- = \eta^2$, and $|\phi_+|^2 = |\phi_-|^2$. So, we can write $\phi_\pm = \eta exp(\pm i\alpha)$ for some constant α, and the vacuum expectation value of the fields $\langle|\phi_\pm|\rangle = \eta$, ie, the vacuum breaks the U(1) gauge symmetry. We can now look for a cylindrically symmetric solution to the model in analogy to the non-supersymmetric case. We write down the ansatz,

$$\begin{aligned}
\phi_0 &= 0 && (19)\\
\phi_+ &= \phi_-^* = \eta e^{in\varphi}f(\rho) && (20)\\
A_\mu &= -\frac{2}{g}n\frac{a(\rho)}{\rho}\delta_\mu^\varphi && (21)\\
F_\pm &= D = 0 && (22)\\
F_0 &= \mu\eta^2(1 - f(\rho)^2). && (23)
\end{aligned}$$

The profile functions $f(\rho)$ and $a(\rho)$ satisfy the following equations:

$$f'' + \frac{f'}{\rho} - n^2\frac{(1 - a(\rho))}{\rho^2} = \mu^2\eta^2(f^2 - 1)f \tag{24}$$

$$a'' - \frac{a'}{\rho} = -g^2\eta^2(1 - a)f^2. \tag{25}$$

Again, the form functions satisfy boundary conditions in which they approach 0 as $\rho \to 0$ and approach 1 as $\rho \to \infty$.

We note the peculiar fact that, at $\rho = 0$, the gauge symmetry is restored as $\phi_\pm \to 0$, but the F_0 term does not vanish. Hence, the string is a region of space with unbroken gauge U(1) symmetry but broken SUSY. Outside of the string, we obtain the ground state in which gauge symmetry is broken but SUSY remains unbroken.

4.3.2 D strings

Now, we can briefly consider the case in which the Fayet-Iliopoulos term is non-vanishing. In this situation, there is only one chiral superfield involved in the symmetry breaking. In order to cancel gauge anomalies, the model must have additional chiral superfields; however, these fields have no effect on symmetry breaking and are invariant under SUSY transformations.[1]

When we set $\kappa = -\frac{1}{2}g\eta^2$, the U(1) symmetry is spontaneously broken under this model. Similarly to the case of F strings, we write down the string solution ansatz,

$$\phi = \eta e^{in\varphi} f(\rho) \tag{26}$$

$$A_\mu = -\frac{2}{g} n \frac{a(\rho)}{\rho} \delta^\varphi_\mu \tag{27}$$

$$D = \frac{1}{2} g\eta^2 (1 - f^2) \tag{28}$$

$$F = 0. \tag{29}$$

The profile functions are now described by first order differential equations

$$f' = n\frac{(1-a)}{\rho} f \tag{30}$$

$$n\frac{a'}{\rho} = \frac{1}{4} g^2\eta^2(1-f^2), \tag{31}$$

along with the same boundary conditions as those of F strings. Note that, like the F strings, the string itself breaks SUSY. However, this time, the breaking comes from a non-vanishing D term at $\rho = 0$, hence the name D strings.

5 Cosmological constraints

Cosmological observations can give strict constraints on the types of topological defects allowed in the Universe. This in turn can give constraints to the allowed theories at high energy scales.

The simplest and perhaps the most stringent constraints come from the observed energy density of the Universe. CMB observations by the Wilkinson Microwave Anisotropy Probe (WMAP) combined with other astronomical observations have determined the total energy density of the Universe to be within a few percent of the critical density.[8] There are other appealing theoretical arguments, such as inflation, that requires the Universe to have total density equal to the critical density. The number density of monopoles produced a standard symmetry breaking is roughly 1 per Hubble volume, so $n_{monopoles} \sim 1/H^{-3} \sim T_c^6/m_p^3$, where T_c is the critical temperature for symmetry breaking and m_p is the Planck mass. The entropy density at this temperature is $\sim T_c^3$, so the monopole to entropy ratio would be $n_{monopoles}/s \simeq 100(T_c/m_p)^3$. Assuming that the entropy is constant (adiabatic processes only), and assuming that the symmetry breaking occurs at temperature $T \sim 10^{14}$GeV, we obtain $n_{monopoles}/s \sim 10^{-13}$, which leads to the present day monopole density roughly $10^1 0$ times bigger than the critical density. Needless to say, this value is in serious conflict with observations today, and monopoles are ruled out with great confidence. We must either have models that do not produce monopoles, or models that produces monopoles before inflation, which would allow the monopole density to be diluted beyond observation. Similar argument can be given for the case of domain walls. In the end, once can show that the energy density of domain walls for typical theories is $\Omega_{DW} \sim (\eta/m_p)^2\eta t$, where η is the energy scale of the phase transition. Using typical values for η, this would imply that the domain wall energy density contribution today would be $\sim 10^{52}$ times the critical density, again far in excess of observation.

5.1 Constraining cosmic strings

Strings are not constrained by the observed geometry of the Universe. After their the formation of strings, their energy density fraction stays constant through the evolution of the Universe. For typical GUT strings (or SUSY GUT strings for that matter), the energy density fraction of strings is $\Omega_{CS} \sim (\eta/m_p)^2 \sim 10^-6$. However, there are various other measurements that constrain the energy density, and hence the energy scale during their formation, of the strings. Below, we discuss some of these constraints.

5.1.1 Constraints from CMB power spectrum

Since cosmic strings possess energy, they are sources of metric deformation (not neccessarily corresponding to Newtonian gravitational attraction) and hence can affect the dynamics of objects and fluids surrounding it. More specifically, their presence can alter the distribution of matter in the early universe and thereby alter the power spectrum of the Cosmic Microwave Background. Comparison of the predicted power spectrum arising from a combination of inflationary scenario and cosmic string and the power spectrum observed by WMAP have ruled out cosmic strings to 90% confidence level.[6] At 99% confidence level, the constraint on the string mass parameter (μ) is $G\mu \leq 1.3 \times 10^{-6}$.

5.1.2 Direct searches for their signature on CMB

One can obtain a comparable constraint on the string mass parameter by directly searching for strings in the CMB anisotropy map. When a string moves across the sky at relativistic velocities, there is a Doppler redshift of light on one side of the string and a blueshift on the opposite side. This effect (known as the Kaiser-Stebbins effect) would show up on the CMB map as an edge discontinuity in the temperature field. Detailed searches for such signatures in the WMAP data have found no matching signal, which implies a constraint of $G\mu \leq 3.3 \times 10^{-7}$.[2, 4]

5.1.3 Other constraints

Constraints on the string mass parameter can be obtained from other observations. For example, one can search for signatures of gravitational waves created by the dynamics os cosmic strings. One can observe the distortions in pulser periodicity caused by gravitational waves to obtain an upper-bound of $G\mu \leq 10^{-7}$.[9]

Another method of cosmic string detection is to look for gravitational lensing events caused by metric deformations around cosmic strings. One candidate for such event is dubbed CSL-1 and was reported by a Russian-Italian collaboration.[7] The detected object is composed of two images of galaxies with identical spectra (to 99.96% confidence) at the same redshift separated by roughly 20 kpc. Extensive modeling of the images show that no traditional matter distribution can result in the configuration of images like that of CSL-1. If one assumes the lens is a cosmic string, one obtains a lower bound on the string mass parameter of $G\mu \geq 4 \times 10^{-7}$, which is consistent with the upper bounds given by other observations.

6 Conclusion and additional remarks

The existence of topological defects were once almost ruled out. Now, there's renewed interest in studying their properties since they may be used to constrain possible higher energy physics theories, such as SUSY. Also, there's nothing better than a good bottle of beer on Friday evenings.

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