MSSM and Dynamical SUSY breaking

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Abstract

In this paper I first give a brief introduction to the Minimal Supersymmetric Standard Model(MSSM). Then I discuss a little bit on why we need supersymmetry, the consequence of electroweak symmetry breaking in MSSM and the mass spectrum. After that, I turn to study dynamical supersymmetry breaking(DSB). I first consider the Supersymmetric QCD(SQCD): its classical moduli spaces, Affine-Dine-Seiberg(ADS) superpotential, and Quantum Moduli Spaces(QMS) when $N_f = N_c$. Then two models, the 3-2 model and Intriligator-Thomas model are proposed as examples on two distinct ways of DSB, which are DSB from ADS criteria and DSB from Deformed QMS, respectively. Finally, I comment on own thoughts on model building towards a visible DSB.

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1 Introduction

2 Supersymmetrizing the Standard Model

2.1 Gerneral SUSY Lagrangians

From our lecture, we know that the most general supersymmetric Lagrangian is[1]:

$$\mathcal{L} = \mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3 = \int d^2\theta \frac{1}{32g^2} tr(W_\alpha W^\alpha)+h.c.+\int d^4\theta \sum_i \Phi_i^\dagger e^{2gV}\Phi_i+\int d^2\theta W(\Phi_i)+h.c.\,. \tag{1}$$

The first term is called gauge kinetic term, the chiral multiplets is constructed from vector multiplets as

$$W_\alpha = \bar{D}^2 e^{-gV} D_\alpha e^{gV}\,. \tag{2}$$

Higher terms of W_α won't appear due to renormalizability. From our homework, we know that expanding the superfield

$$W_\alpha = -i\lambda_\alpha + (\delta_\alpha^\beta D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha^\beta F_{\mu\nu})\theta_\beta + \theta\theta\sigma^\mu_{\alpha\dot{\alpha}} D_m \bar{\lambda}^{\dot{\alpha}}\,. \tag{3}$$

The gauge kinetic term is then written in the component fields of vector multiplets.

$$\mathcal{L}_1 = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{2}D^a D^a - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu \lambda^a\,. \tag{4}$$

The second term is generally called the the Kähler potential, here we consider the simple renormalizable term with trivial metric $g_{IJ^*} = \{+,-,-,-\}$ and $g_{IJ} = 0$. We expand it in its component form as

$$\mathcal{L}_2 = -|D_\mu\phi_i|^2 - i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i + F_i^* F_i + i\sqrt{2}g(\phi_i^* T^a\psi_i\lambda^a - \bar{\lambda}^a T^a\phi_i\bar{\psi}_i) + gD^a\phi_i^* T^a\phi_i\,. \tag{5}$$

[1] We can also put the vector multiplets V in the SUSY D-term, but it is only invariant if the vector multiplets V has $U(1)$ gauge symmetry(the gauge symmetry associated with V can't be non-Abelian. Indeed this term appears in the Fayet-Iliopoulos D-term SUSY breaking.

The last two terms come from the gauge interaction between the vector mutiplets and the chiral mutiplets. Combining Eq.(4) and (5), the equation of motion of the auxiliary D field gives its solution[2]

$$D_a = -g \sum_{ij} \phi_i^* T_a^{ij} \phi_j \,. \tag{6}$$

The last term is the chiral superpotential term, the F-term of the superpotential term is

$$\mathcal{L}_3 = -\frac{1}{2} \frac{\partial^2 W(\phi^k)}{\partial \phi^i \partial \phi^j} \psi^i \psi^j + \frac{\partial W(\phi^k)}{\partial \phi^i} F^i + h.c. \,. \tag{7}$$

Here we write the dependence of superpotential $W(\Phi_i)$ on chiral superfield Φ_i in terms of its scalar part $W(\phi_i)$, which is the same thing. The most general superpotential $W(\Phi_i)$ is

$$W(\Phi_i) = \sum_i \kappa_i \Phi_i + \frac{1}{2} \sum_{i,j} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \sum_{i,j,k} y_{ijk} \Phi_i \Phi_j \Phi_k \,. \tag{8}$$

In the same way, combining Eq.(5), (7) and (8), the equation of motion of the auxiliary F field also gives its solution

$$F_i^* = -\frac{\partial W^\dagger}{\partial \phi^{*i}} = -(\kappa_i + m_{ij}\phi_j + y_{ijk}\phi_j\phi_k) \,. \tag{9}$$

Then we got the relevant components in the superpotential $\mathcal{L}_3$(Here we absorb $F_i^* F_i$ from $\mathcal{L}_2$ into $\mathcal{L}_3$):

$$\mathcal{L}_3 = \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \psi_i^\dagger \psi_j^\dagger - \frac{1}{2} y_{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y_{ijk}^* \phi_i^* \psi_j^\dagger \psi_k^\dagger - V(\phi, \phi^*) \,. \tag{10}$$

The potential term $V(\phi, \phi^*) = F_i^* F_i$ is simply the F-term potential.

The explicitly form of gauge covariant derivatives and field strength are

$$D_m \phi = \partial_m \phi + i g A_m^a T^a \phi \,. \tag{11}$$

[2]The D field would be $D_a = \xi_a - g \sum_{ij} \phi_i^* T_a^{ij} \phi_j$ if there is a Fayet-Iliopoulos (FI) term in the SUSY Lagrangians $\xi V|_D$.

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L \ \ \tilde{d}_L)$	$(u_L \ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\bar{u}$	$\tilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}\ \ \tilde{e}_L)$	$(\nu \ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\bar{e}$	$\tilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u \ \ H^0_u)$	$(\tilde{H}^+_u \ \ \tilde{H}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H^0_d \ \ H^-_d)$	$(\tilde{H}^0_d \ \ \tilde{H}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

Table 1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

$$D_m\psi = \partial_m\psi + igA^a_m T^a\psi\,. \tag{12}$$

$$D_m\lambda^a = \partial_m\lambda^a - gt^{abc}A^b_m\lambda^c\,. \tag{13}$$

$$F^a_{mn} = \partial_m A^a_n - \partial_n A^a_m - gt^{abc}A^b_m A^c_n.\,. \tag{14}$$

2.2 MSSM

In order to minima extend Standard Model, we would naturally think that all the gauge bosons are fitted in the vector supermutiplets while all the chiral supermutiplets are fitted in the chiral supermutiplets. No pair of known particles are in the same supermutiplets, because all the supermutiplets are directly extended from the original Standarde Model particles, their representations of $SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y$ remains the same. The only supermutiplets that share the same representation L is the left handed lepton doublets and

Names	spin 1/2	spin 1	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\tilde{g}$	g	($\mathbf{8}$, $\mathbf{1}$, 0)
winos, W bosons	$\tilde{W}^{\pm}$ $\tilde{W}^{0}$	$W^{\pm}$ W^{0}	($\mathbf{1}$, $\mathbf{3}$, 0)
bino, B boson	$\tilde{B}^{0}$	B^{0}	($\mathbf{1}$, $\mathbf{1}$, 0)

Table 2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Higgs chiral supermutiplets H_d. However, this would introduce two major problems. First we drop one supermutiplet so that anomaly cancellation does not hold any more. Second the Higgs will also carry the lepton number so that lepton number is not a conserved quantity any more.

There are three sufficient reasons why we need two Higgs doublets: The first is that we can not construct H_d from $H_d^{\alpha} = \epsilon_{\alpha\beta} H_u^{\beta}$ because H_u are in the chiral supermutiplets and they are holomorphic. The second is that the anomaly cancellation requires two Higgs doublets. The third one, as I will show it in hte following section, is that the supersymmetric grand unification needs two Higgs doublets.

After signing the gauge transformation property to all the superfields, we could see that the first two terms in the general SUSY Lagrangian is *fixed*, except the $F_i^* F_i$ term. The gauge kinetic terms $\mathcal{L}_1$ give us the kinetic energy for the gauge boson and its SUSY partner-gaugino, and their interaction through covariant derivative. The kähler potential gives us kinetic energy for the chiral components: fermion, sfermion, Higgs and Higgsinos, their interaction with gauge fields through covariant derivatives and scalar-fermion-gaugino interaction term $\phi^* \psi \lambda$. The only nontrival construction for MSSM before SUSY breaking and electroweak breaking is the superpotential form[3]. Notice that there is no gauge singlet in the chiral supermutiplets, there must be no linear term in the superpotential since we can not make it uncharged under gauge symmetry[1]. Notice also that there is *no mass* terms in $\mathcal{L}_1$ and $\mathcal{L}_2$(We must have a nonzero, actually, negative mass term for the Higgs boson) and we must give masses to the fermions aftre SUSY is broken. Naturally, we would guess the superpotential of MSSM is given by

$$W_{MSSM} = y_{ij}^{(d)} Q_{i\alpha}^a \bar{d}_{ja} H_d^\alpha + y_{ij}^{(u)} Q_{i\alpha}^a \bar{u}_{ja} H_u^\alpha + y_{ij}^{(e)} L_{i\alpha} \bar{e}_j H_d^\alpha + \mu H_{u\alpha} H_d^\alpha . , \tag{15}$$

which automatically maintain many accidental symmetries in the SM, like Baryon, lepton number. The $\alpha = 1, 2$ is the $SU(2)_L$ weak isospin index, the doublets is then tied by a $\epsilon_{\alpha\beta}$ in a gauge-invariant way. $i = 1, 2, 3$ is the flavor or family index, and $a = 1, 2, 3$ is the color index. We will suppress all the gauge index by writing $y_{ij}^{(d)} Q_{i\alpha}^a \bar{d}_{ja} H_d^\alpha$ into $y_{ij}^{(d)} Q_i \bar{d}_j H_d$. Besides the terms we needed here, we can not forbid terms like $Q\bar{d}L$, $H_u L$ due to renormalizability. But direct violation of such accidental symmetry will cause rapid proton decay due to dimension five operators $\bar{u}, \bar{d} \longleftrightarrow QL$ mediated by the squark. So we introduce R parity $P_R = (-1)^{3(B-L)+2s}$[2]. Such additional discrete symmetry also guarantee that the lightest particles in each different charges is stable, which is just like the case that lepton number conservation make e^- stable. If the new stable particle with $P_R = -1$ from R-parity, called "lightest SUSY particel" or LSP, is neutral, it only interacts weakly with ordinary matter through high-dimension operator which integrates out the R-parity violation at UV and makes a perfect candidate for dark matter around TeV.

[3]Actually, I guess that is why in almost all the paper about MSSM, their first introduction is always $W_{MSSM} = ...$

2.3 Guidance Principle to SUSY model

The first and perhaps the biggest motivation to supersymmetrize the Standard Model is to solve the gauge hierarchy problem(GHP): In the Standard Model, the Higgs potential is given by

$$V = \mu^2 |H|^2 + \lambda |H|^4, \tag{16}$$

where $v^2 = \langle H \rangle^2 = -\mu^2/2\lambda = (176 \text{ GeV})^2$. Because perturbative unitarity requires that $\lambda \lesssim 1$, $-\mu^2$ is of the order of $(100 \text{ GeV})^2$. However, the mass squared parameter μ^2 of the Higgs doublet receives a *quadratically* divergent contribution from its self-energy corrections. For instance, the process where the Higgs doublets splits into a pair of top quarks and come back to the Higgs boson gives the self-energy correction

$$\Delta\mu^2_{\text{top}} = -6\frac{h_t^2}{4\pi^2}\Lambda^2_{UV}, \tag{17}$$

Since $\Delta\mu^2_{\text{top}} \gg \mu^2$ if we want to apply the Standard Model to a very high-energy scale, we would conclude that the "bare" μ must be positive and extremely finely-tunned so that they give a large cancellation to get the physical μ, which is around electroweak scale. This indicates the Standard Model does not applicable below the distance scale of 10^{-17} cm.

The motivation for supersymmetry is to make the Standard Model applicable to much shorter distances so that we can answers many puzzles in the Standard Model by requiring new physics at shorter distance scales. In order to do so, supersymmetry repeats what history did with the gauge symmetry and the vacuum polarization: doubling the degrees of freedom with an explicitly broken new symmetry. Then the top quark would have a superpartner, stop, whose loop diagram gives another contribution to the Higgs boson self energy

$$\Delta\mu^2_{\text{stop}} = +6\frac{h_t^2}{4\pi^2}\Lambda^2_{UV}. \tag{18}$$

The leading pieces in $1/r_H$ cancel between the top and stop contributions, and one obtains the correction to be

$$\Delta\mu^2_{\text{top}} + \Delta\mu^2_{\text{top}} = -6\frac{h_t^2}{4\pi^2}(m_{\tilde{t}}^2 - m_t^2)\log\frac{\Lambda_{UV}^2}{m_{\tilde{t}}^2}. \tag{19}$$

One important difference from the positron case, however, is that the mass of the stop, $m_{\tilde{t}}$, is unknown. In order for the $\Delta\mu^2$ to be of the same order of magnitude as the tree-level value $\mu^2 = -2\lambda v^2$, we need $m_{\tilde{t}}^2$ to be not too far above the electroweak scale. Similar arguments apply to masses of other superpartners that couple directly to the Higgs doublet. This is the so-called naturalness constraint on the superparticle masses.

However, such constraints introduce a new problem: Why SUSY breaking scale is much lower than the fundamental scale? Where is such SUSY breaking scale come from even in the absence of killing the quadratical divergence. There is usually called the "μ" problems. In a SUSY theory, such puzzles is expected to be solved that SUSY is broken *dynamically*. The large hierarchy between the SUSY breaking scale and the fundamental scale is generated by the non-perturbative effects suppressed by $e^{-8\pi^2/g^2}$ that could break supersymmetry. This is perfectly similar to the QCD and superconductivity. The QCD scale or the critical temperature is a derived energy scale from the initial coupling constant at the ultraviolet scale or the typical energy of the free electrons which is connected by an exponential function.

So personally, I would rather to divide the GHP in two related problems:

- Where does the small ratio of scale $m_{3/2}/\Lambda_{UV}$ or μ/Λ_{UV} come from?
- How could we kill the quadratical divergence for the Higgs boson masses?

Note that there are also two other main different solutions to the GHP, one is technicolor and little Higgs. Both of them derive the smallness of TeV from the fundamental scale dynamically. The technicolor states that there is simply no Higgs boson while we still could maintain electroweak symmetry breaking similarly as the Higgs mechanism, while the little Higgs kill the quadratical divergence for the Higgs boson at the leading loop corrections by introducing a similar partner of the SM particles in the same statistics. Another different way is to solve GHP is that the fundamental scale in 4D is TeV large introducing large extra dimensions, which answer the two problems at the same time, and the smallness of the 4D fundamental scale is answered dynamically by Randall-Sundrum model summing that the extra dimension is exponentially wrapped.

If we believe Grand Unification at very high-energy, which is, the three gauge forces should form a unified structure and the distinction between quarks and leptons should disappear, then we will see that indeed, the coupling strength will unify at very high-energy simply from running of the Renormalization Group(RG) equation in the perturbative regime.

The fact that there is a single gauge coupling constant fro a simple GUT group G means the quantity $g_i^2 \mathrm{Tr}(T_i^2)$ should be the same at the unification scale m_X. Here, the index $i = 1, 2, 3$ labels the standard model group U(1), SU(2), SU(3). By playing the algebra a little bit, we will obtain the ratio between the couplings strength $\alpha_i \equiv g_i^2/4\pi$ and fine-constant α at the unification scale:

$$\begin{aligned} \sin^2\theta_W = \frac{\alpha}{\alpha_2} = \frac{3}{8} \\ \alpha_1 = \frac{5}{8}\alpha \\ \alpha_2 = \alpha_3 = \frac{3}{8}\alpha = \alpha_u\,, \end{aligned} \tag{20}$$

where θ_W is the Weinberg angle. We redefine $\alpha_1 \to \frac{5}{3}\tilde{\alpha}_1$ so that it also equals the unification coupling α_u at m_X.

The one-loop RG equation is a simple analytic function and we have the result

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_u} + \frac{b_i}{2\pi} \log\left(\frac{m_X}{\mu}\right) . \tag{21}$$

The b_i are pure numbers determined by the particle quantum numbers and turned out

	SM	MSSM
b_1	$\frac{4}{3}n_g + \frac{1}{10}n_h (41/10)$	$2n_g + \frac{3}{10}n_h$ $(33/5)$
b_2	$\frac{4}{3}n_g + \frac{1}{6}n_h - \frac{22}{3}$ $(-19/6)$	$2n_g + \frac{1}{2}n_h - 6$ (1)
b_3	$\frac{4}{3}n_g - 11$ (-7)	$2n_g - 9$ (-3)

Table 3: One-loop beta function parameters for the three gauge groups in the standard model and the MSSM for the case of n_g families and n_h Higgs doublets. The value in the parentheses are the standard value for $n_g = 3$, $n_h = 1$ for standard model and $n_h = 2$ for the MSSM.

to be as given in Table. 3. The symbols n_g and n_h represent the number of generations of quarks and leptons and the number of Higgs doublets. Putting in the standard values for the number of generations and the number of Higgs doublets gives the results shown in Table. 3. The most significant difference between the standard model and the MSSM is the shift in b_3 due to gluino contributions.

In order to solve for α_u and m_X in terms of measured $\alpha_i(m_Z)$, we have three equations for two unknowns. So there is a relation among the α_i's that is required;

$$\frac{\alpha_1^{-1} - \alpha_2^{-1}}{\alpha_2^{-1} - \alpha_3^{-1}} = \frac{b_1 - b_2}{b_2 - b_3} . \tag{22}$$

Note that α_i's are functions of energy, but this combination is predicted to be constant. Experimentally, the numbers are best measured for $\mu = m_Z$ (in study of Z decay). The results are $\sin^2\theta_W = 0.232$, $\alpha_{em}^{-1}(m_Z) = 128$, $\alpha_1^{-1}(m_Z) = 59.0$, $\alpha_2^{-1}(m_Z) = 29.7$. The errors are dominated by $\alpha_2^{-1}(m_Z) = 8.5 \pm 0.3$. So experimentally

$$\frac{\alpha_1^{-1} - \alpha_2^{-1}}{\alpha_2^{-1} - \alpha_3^{-1}} = 1.38 \pm 0.02 \,. \tag{23}$$

Using numbers in the Table. 3, we see that at this order of approximation the value of this independent of the number of generations. Also it is 2 for both standard model and the MSSM if there are no Higgs doublets and it decreases as Higgs doublets are added. Putting the standard value gives the results:

$$\begin{aligned} \text{SM}: \frac{b_1 - b_2}{b_2 - b_3} &= 1.90 \\ \text{MSSM}: \frac{b_1 - b_2}{b_2 - b_3} &= 1.40 \,. \end{aligned} \tag{24}$$

Comparing to the experimental value, the MSSM predicions is highly preferred over the standard model. More refined analysis have been carried out that

- Include the top quark contribution.
- Evolve from m_Z to $m_{3/2}$(which is allowed to vary) using the two-loop standard model equations.
- Include threshold effects at $m_{3/2}$ to properly decribed the turning on of SUSY partner contribution.
- Evolve from $m_{3/2}$ to m_X using the two loop MSSM equation.

We may ask the question, why the soft SUSY breaking terms do not destroy the fact of SUSY GUT, since all the result are actually from the MSSM without soft SUSY breaking terms. The truth is that just like Yukawa couplings. Such trilinear couplings comes into the running of the gauge couplings at two loop level. All the two loop diagram does not have divergent terms at all, so they don't have any contributions to the running of the gauge coupling. Actually, there is a general theorem that all the higher dimension couplings does not have a contribution to the running of the RG equation of the lower dimension couplings. We could also see in the previous section, the trilinear couplings come into the running of the mass couplings which confirm such theorem.

The third reason that why we need SUSY is that we want LSP as a WIMP, which is a perfect dark matter candidate. The usual LSP in SUSY is neutralino or gravitino, depending different SUSY breaking mechanism. As the universe expands, they drop out of thermal equilibrium and decoupled when the annihilation rate becomes comparable to the expansion rate. This allows one to compute the residual density as a function of the neutralino mass. One finds that the contribution

$$\Omega_{LSP} \equiv \text{density/critical density}\,, \tag{25}$$

is proportional to the mass, approaching unity for $M_{LSP} \sim 200\text{GeV}$. The precise value is model dependent. For neutralino, it depends on exactly what mixture of gauginos and higgsinos it contains. The desired value of Ω_{LSP} required to account for dark matter is about 0.3, so the mass is likely to below 100GeV.

2.4 Electroweak Breaking in a Soft SUSY Breaking Phase

From a bottom-up point of view, we could simply ignore the question how to break SUSY and see what happened in TeV scale and its implications to the extension of SM and cosmology. We then write the term *by hand* to lift the degeneracy and constraints due to SUSY. The basic rules are from the following:

- In order to maintain the fact that SUSY is one of the solution to GHP. Such explicitly SUSY breaking terms should not introduce new power divergence to the Higgs masses, more broadly, any scalar masses that their quadratic divergence can not be killed by gauge invariance. The criteria for such terms is "soft", which means they have dimensionful parameters. The reason is that the only power divergence Feymann diagram are fermion self-energy diagram and scalar vacuum polarization diagram. Both are forbidden if the interaction vertex are dimensionful in the diagram(The external lines for the two must be at most dimension 1/2 or 0, respectively). A simple example is to consider soft terms in the interaction from the superpotential $W = hQ\bar{u}H_u$:

$$\mathcal{L} \supset -m_Q^2|\tilde{Q}|^2 - m_{\bar{u}}^2|\tilde{\bar{u}}|^2 - hA\tilde{Q}\tilde{\bar{u}}H_u \,, \tag{26}$$

With a simplifying assumption $m_Q^2 = m_{\bar{u}}^2 = \tilde{m}^2$, we find

$$\delta m_H^2 = -\frac{6h^2}{(4\pi)^2}\tilde{m}^2 \log\frac{\Lambda^2}{\tilde{m}^2} \,, \tag{27}$$

- We know electroweak symmetry is spontaneously broken, so we can not write soft mass terms for Standard Model particles except Higgs. We will see below that only μ mass term can not give a negative mass term to make spontaneous electroweak symmetry breaking, so we need the soft mass term for Higgs and especially its off-diagonal mass term $B\mu H_u H_d$. We also have to give a heavy mass term for the sparticles(there is no mass term for sparticles in W if we don't have a linear term in superpotential, like MSSM). Other soft term contains three scalar trilinear terms $a^{ijk}\phi_i\phi_j\phi_k$ and the bilinear terms $b^{ij}\phi_i\phi_j$. The trilinear singlets coupling $a_i^{jk}\phi^{*i}\phi_j\phi_k$ can lead to quadratic divergences if any of the chiral supermutiplets are gauge singlet despite the fact that they are formally soft.

With generic soft SUSY breaking terms, the relevant Lagrangian to the Higgs sector of MSSM is written as

$$\begin{aligned} V &= \frac{g_1^2}{2}\Big(\bar{H}_u\frac{1}{2}H_u + \bar{H}_d\frac{-1}{2}H_d\Big)^2 + \frac{g_2^2}{2}\Big(\bar{H}_u\frac{\vec{\sigma}}{2}H_u + \bar{H}_d\frac{\vec{\sigma}}{2}H_d\Big)^2 + \mu^2(|H_u|^2 + |H_d|^2) \\ &+ m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 - (B\mu H_u H_d + c.c.) \end{aligned} \tag{28}$$

The first two terms are from the D-term Higgs potential, the third term is from the F-term superpotential $\mu H_u H_d$, and last term are explicitly the soft SUSY breaking term.

If we assume that only the neutral components have vacuum expectation values which conserves $U(1)_{em}$[4], we have

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ \nu_u \end{pmatrix}, \qquad \langle H_d \rangle = \begin{pmatrix} \nu_d \\ 0 \end{pmatrix}, \tag{29}$$

in the vacuum. Writing the potential (28) down using the expectation values (29), we find

$$V = \frac{g_Z^2}{8}(\nu_u^2 - \nu_d^2)^2 + \begin{pmatrix} \nu_u & \nu_d \end{pmatrix} \begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix} \begin{pmatrix} \nu_u \\ \nu_d \end{pmatrix}, \tag{30}$$

where $g_Z^2 = g_1^2 + g_2^2$. In t=oder for the Higgs bosons to acquire the vacuum expectation values, the determinant of the mass matrix at the origin must be negative,

$$\det \begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix} < 0\,. \tag{31}$$

From the Higgs mechanism, the W and Z masses in the tree level are given by ν^2, just as in the SM

$$m_W^2 = \frac{1}{4}g_2^2\nu^2 \qquad m_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)\nu^2\,, \tag{32}$$

[4]This is not necessarily true in general two-doublet Higgs model. Consider a review, see "Higgs Hunter's Guide".

where we define

$$\nu_u = \frac{\nu}{\sqrt{2}}\sin\beta, \quad \nu_d = \frac{\nu}{\sqrt{2}}\cos\beta, \quad \nu = 250\text{GeV}\,. \tag{33}$$

to reproduce the mass of W and Z bosons correctly. The vacuum minization condition are given by $\partial V/\partial\nu_u = \partial V/\partial\nu_d = 0$ from the potential Eq.(30). Using Eq.(33), we obtain

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1}\,, \tag{34}$$

and

$$B\mu = (2\mu^2 + m_{H_u}^2 + m_{H_d}^2)\sin\beta\cos\beta\,. \tag{35}$$

There are two Higgs doublets, each of which contains four real degree of freedom. After electroweak symmetry breaking, three of them are eaten by $W^\pm$ and Z bosons, and we are left with five physics scalar particles. They are two CP-even neutral scalars h, H; one CP-odd scalar A, and two charged scalars H^+ and H^-. If we define

$$m_A^2 = 2\mu^2 + m_{H_u}^2 + m_{H_d}^2\,, \tag{36}$$

which happens to be the mass of the CP-odd scalar A, we can simply solve the parameter μ, m_{H_u}, and m_{H_d} in term of m_A, m_Z, and β. We then write the mass matrix of two CP-even neutral scalars h, H, which corresponds to the mass eigenstates of pair of fields $\phi_u \equiv \text{Re}H_u^0$, and $\phi_d \equiv \text{Re}H_d^0$, into

$$\begin{pmatrix} m_Z^2(\sin\beta)^2 + m_A^2(\cos\beta)^2 & -(m_A^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_A^2 + m_Z^2)\sin\beta\cos\beta & m_Z^2(\cos\beta)^2 + m_A^2(\sin\beta)^2 \end{pmatrix}. \tag{37}$$

The invariance of the trace and the determinant when diagonalizing the mass matrix $m_h^2 + m_H^2 = m_A^2 + m_Z^2$, $m_h^2 m_H^2 = m_A^2 m_Z^2\cos^2 2\beta$ tells us that

$$m_{h,H}^2 = \frac{1}{2}\Big(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2\cos^2\beta}\Big)\,. \tag{38}$$

In a similar way, we could get the masses for the charged Higgs boson $H^{\pm}$ are

$$m_{H^{\pm}}^2 = m_W^2 + m_A^2\,. \tag{39}$$

We can see that the tree level mass boundary of the light CP-even neutral Higgs h is

$$m_h \leq m_Z|\cos 2\beta|\,. \tag{40}$$

when $m_A \to \infty$. Since SUSY is softly broken, the tree level boundary Eq.(40) can be pushed up to 130GeV by the one-loop level correction

$$\Delta(m_h^2) = \frac{N_c}{4\pi^2} h_t^2 \nu^2 \sin^4\beta \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_{\tilde{t}}^2}\right) \tag{41}$$

if the scalar top mass is up to 1TeV.

Once the electroweak symmetry is broken, and since SUSY is explicitly broken by soft terms in MSSM, there is no quantum number which can distinguish two neutral higgsino states $\tilde{H}_u^0$, $\tilde{H}_d^0$, and two neutral gaugino states $\tilde{W}^3$ (neutral wino) and $\tilde{B}$ (bino). They have a four-by-four Majorana mass matrix

$$\mathcal{L} \supset -\frac{1}{2}\begin{pmatrix} \tilde{B} & \tilde{W}^3 & \tilde{H}_d^0 & \tilde{H}_u^0 \end{pmatrix} \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & -m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix}. \tag{42}$$

Here $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$. The diagonal elements M_1 and M_2 are the soft Majorana mass term for gaugino, μ term in the superpotential contributes a Dirac mass term for the Higgsino. The mixing terms between neutral Wino, Bino and the Higgsinos came from the gaugino-fermion-scalar coupling term

$i\sqrt{2}g(\phi_i^* T^a \psi_i \lambda^a - \bar{\lambda}^a T^a \phi_i \bar{\psi}_i)$ in the Kähler potential in Eq.(5). Once M_1, M_2, μ exceed m_Z, which is preferred given the current experimental limits, one can regard components proportional to m_Z as small perturbations. Then neutralino mass eigenstates are very nearly $\tilde{N}_1 \approx \tilde{B}$; $\tilde{N}_2 \approx \tilde{W}^0$; $\tilde{N}_3, \tilde{N}_4 \approx \tilde{H}_u^0 \pm \tilde{H}_d^0/\sqrt{2}$.

Similarly two positively charged inos: $\tilde{H}_u^+$ and $\tilde{W}^+$, and two negatively charged inos: $\tilde{H}_d^-$ and $\tilde{W}^-$ mix. The mass matrix is given by

$$\mathcal{L} \supset -\begin{pmatrix} \tilde{W}^- & \tilde{H}_d^- \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} + c.c.\,. \tag{43}$$

Again once $M_2, \mu \geq m_W$, the charginos states are close to the weak eigenstates winos and higgsinos.

2.5 Running of the Soft Parameter and Radiative Electroweak Symmetry Breaking

The soft parameters are somewhat arbitrary in the low energy. However, we can use the renormalization group equation from boundary conditions at high energy suggested by different SUSY breaking models to obtain useful information on the "typical" superparticle mass spectrum.

here I simply figure out some important future for such running of the soft parameters. First, the gaugino mass parameters have a very simply behavior that

$$\frac{d}{d \ln \Lambda} \frac{M_i}{g_i^2} = 0\,. \tag{44}$$

Therefore, the ratio M_i/g_i^2 are constant at all energies. If grand unification is true, both the gauge coupling constnats and gaugino mass paramters must unify at the GUT-scale and hence the low energy gaugino mass ratios are predicted to be[5]

$$M_1 : M_2 : M_3 = g_1^2 : g_2^2 : g_3^2 \sim 1 : 2 : 7 \tag{45}$$

at TeV scale. we see that the tendency that colored particle (gluino in this case) is much heavier than uncolored particle (wino and bino in this case). This turns out be a relatively model-independent conclusion.

In order to reproduce the SM Lagrangian properly, a negative mass squared in the Higgs potential is required. In the last section, we know that a off-diagonal soft Higgs mass term $B\mu H_u H_d$ will give a negative physical Higgs mass. Actually, in principle, running of renormalization group will give a nonzero negative physical Higgs mass even in absence of a off-diagonal soft Higgs mass term but not the squarks and sleptons. This is very interesting in a sense that it answers why Higgs is the only scalar fields that condense, because in MSSM, there are so many scalar fields and Higgs is only one of them.

The running of the scalar masses is given by simply equations when all Yukawa couplings other than that of the top quarks are neglected. We find

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{H_u}^2 = 3X_t - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 \,, \tag{46}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{H_d}^2 = -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 \,, \tag{47}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{Q_3}^2 = X_t - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 \,, \tag{48}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{\bar{u}_3}^2 = 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 \,, \tag{49}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{\bar{d}_3}^2 = -\frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 \,, \tag{50}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{L_3}^2 = -6g_2^2 M_2^2 - \frac{3}{5} g_1^2 M_1^2 \,, \tag{51}$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{\bar{e}_3}^2 = -\frac{24}{5} g_1^2 M_1^2 \,. \tag{52}$$

[5]This result is violated by anomaly-mediated SUSY breaking.

Here $X_t = 2h_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\tilde{u}_3}^2 + A_t^2)$, A_t is the trilinear couplings. We can see that gauge interactions push the scalar masses up at lower energy due to the gaugino mass squared contributions and Yukawa couplings push the scalar masses down at lower energies. From Eq.(45), we can see that $g_3^2 M_3^2 \gg g_2^2 M_2^2$ and $g_1^2 M_1^2$. It is extremely interesting to realize that $m_{H_u}^2$ is pushed down the most because of the factor 3 as well as the absence of gluino mass contribution. This provide a compelling solution that $m_{H_u}^2$ is running to negative at low energy (1TeV) and why *only* Higgs boson get a negative mass-squared and condenses.

3 Dynamical SUSY Breaking

There are two distinct ways to break SUSY dynamically. The old one has been suggested by Affleck, Dine, and Seiberg (ADS) in Refs. [4, 5]. It relies on two basic requirements: The first one is that there be no non-compact flat directions in the classical scalar potential; the second one is that there exist a spontaneously broken global symmetry which gives a *massless* Goldstone boson, and unbroken SUSY leads to an additional massless scalar partner to complete the supermultiplet(a modulus), but if there are no flat directions this is impossible[6]. We will refer to this condition for DSB as to the ADS criterion. In the early days people looked for theories that had no classical flat directions and tried to make them break global symmetries in the perturbative regime. This method produced a handful of dynamical SUSY breaking theories. Now with duality we can find many examples of dynamical SUSY breaking. An important twist is that we will find that non-perturbative quantum effects can lift flat directions both at the origin of moduli space[14, 15] as well as for large VEVs.

[6]In special cases, the extra massless scalar could be a Goldstone boson itself, thus evading the conclusion of non-compact flat directions and, ultimately, of supersymmetry breaking.

3.1 Non-perturbative Corrections to Superpotential

We first consider the holomorphic gauge coupling

$$\begin{aligned}\mathcal{L} &= \frac{1}{16\pi i}\int d^4x \int d^2\theta \tau W^a_\alpha W^a_\alpha + h.c. \\ &= \int d^4x\Big[-\frac{1}{4g^2}F^{a\mu\nu}F^a_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2}F^{a\mu\nu}\tilde{F}^a_{\mu\nu} + \frac{i}{g^2}\lambda^{a\dagger}\bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2g^2}D^a D^a\Big] , \end{aligned} \quad (53)$$

where

$$\tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^a_{\alpha\beta} , \quad (54)$$

$$\tau \equiv \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2} , \quad (55)$$

The one-loop running of the gauge coupling g is given by the renormalization group equation:

$$\mu\frac{dg}{\mu} = -\frac{b}{16\pi^2}g^3 , \quad (56)$$

where for an $SU(N)$ gauge theory with F flavor and N=1 supersymmetry

$$b = 3N_c - N_f , \quad (57)$$

The solution for the running coupling is

$$\frac{1}{g^2(\mu)} = -\frac{b}{8\pi^2}\ln\left(\frac{|\Lambda|}{\mu}\right) . \quad (58)$$

We can then absorb the θ_{YM} term by making the energy cutoff complex:

$$\tau_{1-\text{loop}} = \frac{\theta_{\text{YM}}}{2\pi} + \frac{4\pi i}{g^2(\mu)} \quad (59)$$

$$= \frac{1}{2\pi i}\ln\left[\left(\frac{|\Lambda|}{\mu}\right)^b e^{i\theta_{\text{YM}}}\right] . \quad (60)$$

We can then define a holomorphic intrinsic scale

$$\Lambda \equiv |\Lambda| e^{i\theta_{\text{YM}}/b} = \mu e^{2\pi i\tau/b} , \quad (61)$$

or equivalently

$$\tau_{1-\text{loop}} = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) \ . \tag{62}$$

The θ_{YM} angle term which violate CP is a total derivative and have no effects in perturbation. The non-perturbative effects can be see by considering a semi-classical instanton configuration of gauge field.

$$A^a_\mu(x) = \frac{h^a_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2}, \tag{63}$$

where $h^a_{\mu\nu}$ describes how the instanton is oriented in the gauge space and spacetime. Equation (63) represents an instanton configuration of size ρ centered about the point x_0^ν. Such instantons have a non-trivial, topological winding number, n, which takes integer values. The CP violating term measures the winding number:

$$\frac{\theta_{\rm YM}}{32\pi^2}\int d^4x\, F^{a\mu\nu}\tilde{F}^a_{\mu\nu} = n\,\theta_{\rm YM} \ . \tag{64}$$

Since the action appears exponentially in the path integral e^{iS} and it depends on θ_{YM} only through a term that is an integer times θ_{YM}, it follows that

$$\theta_{YM} \to \theta_{YM} + 2\pi \ , \tag{65}$$

is a symmetry of the theory since it has no efects on the path integral.

If we integrate down to the scale μ we have the effective superpotential

$$W_{eff} = \frac{\tau(\Lambda;\mu)}{16\pi i} W^a_\alpha W^a_\alpha \ . \tag{66}$$

To allow for non-perturbative corrections we can write the most general form of τ as:

$$\tau(\Lambda;\mu) = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda;\mu) \ , \tag{67}$$

where f is a holomorphic function of Λ. Since $\Lambda \to 0$ corresponds to weak coupling where we must recover the perturbative result (62), f must have Taylor series representation in positive powers of Λ. The instanton corrections f must be constrained by symmetry $\Lambda \to e^{2\pi i/b}\Lambda$, so the Taylor series must be in positive powers of Λ^b. The reason that it is in positive powers of Λ^b instead of Λ is that the typical one instanton effects are suppressed by

$$e^{-S_{\rm int}} = e^{\frac{-8\pi^2}{g^2(\mu)}+i\theta_{\rm YM}} = \left(\frac{\Lambda}{\mu}\right)^b . \tag{68}$$

Thus in general we can write:

$$\tau(\Lambda;\mu) = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu}\right)^{bn} \tag{69}$$

3.2 The Classical Moduli Space of Supersymmetric QCD(SQCD)

Consider $SU(N_c)$ SUSY QCD with N_f flavors. This theory has a global $SU(N_f)\times SU(N_f)\times U(1)\times U(1)_R$ symmetry. The quantum numbers[7] of the squarks and quarks are summarized below, where $\Box$ denotes the fundamental representation of the group:

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)$	$U(1)_R$
Φ, Q	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{N_f-N_c}{N_f}$
$\overline{\Phi}$, $\overline{Q}$	$\overline{\Box}$	$\mathbf{1}$	$\overline{\Box}$	-1	$\frac{N_f-N_c}{N_f}$

(70)

The $SU(N_f)\times SU(N_f)$ global symmetry is the analog of the $SU(3)_L\times SU(3)_R$ chiral symmetry of non-supersymmetric QCD with 3 flavors, while the $U(1)$ is the analog[8] of baryon number since quarks (fermions in the fundamental representation of the gauge group) and anti-quarks (fermions in the anti-fundamental representation of the gauge group) have opposite charges. There is an additional $U(1)_R$ relative to non-supersymmetric QCD since in the supersymmetric theory there is also a gaugino. In the following discussions, we just

[7]As usual only the R-charge of the squark is given, and $R[Q] = R[\Phi] - 1$.

[8]Up to a factor of N_c.

consider the case for $N_f \geq N_c$ in the matrix notations, the case for $N_f < N_c$ should be similar and will be discussed in the following section.

Recall that the D-terms for this theory are given in terms of the squarks by

$$D^a = g(\Phi^{*jn}(T^a)_n^m \Phi_{mj} - \overline{\Phi}^{jn}(T^a)_n^m \overline{\Phi}^*_{mj}) \ , \tag{71}$$

where j is a flavor index that runs from 1 to N_f, m and n are color indices that run from 1 to N_c, the index a labels an element of the adjoint representation, running from 1 to $N_c^2 - 1$, and T^a is a gauge group generator. The D-term potential is:

$$V = \frac{1}{2g^2} D^a D^a \ , \tag{72}$$

where we sum over the index a.

Define

$$D_m^n \equiv \langle \Phi^{*jn} \Phi_{mj} \rangle \ , \tag{73}$$

$$\overline{D}_m^n \equiv \langle \overline{\Phi}^{jn} \overline{\Phi}^*_{mj} \rangle \ . \tag{74}$$

D_m^n and $\overline{D}_m^n$ are $N_c \times N_c$ positive semi-definite Hermitian matrices. In a SUSY vacuum state the vacuum energy vanishes and we must have:

$$D^a \equiv g T_n^{am}(D_m^n - \overline{D}_m^n) = 0 \ . \tag{75}$$

Since T^a is a complete basis for traceless matrices, we must have that the second matrix is proportional to the identity:

$$D_m^n - \overline{D}_m^n = \rho I \ . \tag{76}$$

D_m^n can be diagonalized by an $SU(N_c)$ gauge transformation

$$D' = U^\dagger D U \ , \tag{77}$$

so we can take D^n_m to have the form:

$$D = \begin{pmatrix} |v_1|^2 & & & \\ & |v_2|^2 & & \\ & & \ddots & \\ & & & |v_N|^2 \end{pmatrix} . \tag{78}$$

In this basis, because of Eq. (76), $\overline{D}^n_m$ must also be diagonal, with eigenvalues $|\overline{v}_i|^2$. This tells us that

$$|v_i|^2 = |\overline{v}_i|^2 + \rho \ . \tag{79}$$

Since D^n_m and $\overline{D}^n_m$ are invariant under flavor transformations, we can use $SU(N_f)\times SU(N_f)$ flavor transformations to put $\langle\Phi\rangle$ and $\langle\overline{\Phi}\rangle$ in the form

$$\langle\Phi\rangle = \begin{pmatrix} v_1 & & & 0 & \ldots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \ldots & 0 \end{pmatrix} , \quad \langle\overline{\Phi}\rangle = \begin{pmatrix} \overline{v}_1 & & \\ & \ddots & \\ & & \overline{v}_N \\ 0 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & 0 \end{pmatrix} . \tag{80}$$

Thus we have a space of degenerate vacua, which is referred to as a moduli space of vacua. The vacua are physically distinct since, for example, different values of the VEVs correspond to different masses for the gauge bosons.

With a VEV for a single flavor turned on we break the gauge symmetry down to $SU(N_c-1)$. At a generic point in the moduli space the $SU(N_c)$ gauge symmetry is broken completely and there are $2N_cN_f-(N_c^2-1)$ massless chiral supermultiplets left over. We can describe these light degrees of freedom in a gauge invariant way by scalar "meson" and "baryon" fields and their superpartners:

$$M_i^j = \overline{\Phi}^{jn}\Phi_{ni} \ , \tag{81}$$

$$B_{i_1,\ldots,i_N} = \Phi_{n_1i_1}\ldots\Phi_{n_Ni_N}\epsilon^{n_1,\ldots,n_N} \ , \tag{82}$$

$$\overline{B}^{i_1,\ldots,i_N} = \overline{\Phi}^{n_1i_1}\ldots\overline{\Phi}^{n_Ni_N}\epsilon_{n_1,\ldots,n_N} \ . \tag{83}$$

The fermion partners of these fields are the corresponding products of scalars and one fermion. At the classical level, for $N_c=N_f$, there is a relationship between the product of the B and $\overline{B}$ eigenvalues and the product of the non-zero eigenvalues of M due to the contraction of the antisymmetric epsilon tensors[9]:

$$B_{i_1,\ldots,i_N}\overline{B}^{j_1,\ldots,j_N} = N_c!M^{j_1}_{[i_1}\ldots M^{j_N}_{i_N]} \ , \tag{84}$$

where [ ] denotes antisymmetrization. We can also write it into a more compact form:

$$B\bar{B}-\mathrm{det}M = 0 \ . \tag{85}$$

. This constraint could also be understood as a trivial consequence of the Bose statistics of the underlying theory. For a generic constraints from the Bose statistics of the fundamental quarks, see Ref. [13].

[9] This equation in John Terner's note is wrong as he miss a factor of N_c!

Up to flavor transformations the moduli can be written as:

$$\langle M \rangle = \begin{pmatrix} v_1\overline{v}_1 & & & & & \\ & \ddots & & & & \\ & & v_N\overline{v}_N & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}, \qquad (86)$$

$$\langle B_{1,\ldots,N} \rangle = v_1 \ldots v_N \ , \qquad (87)$$

$$\langle \overline{B}^{1,\ldots,N} \rangle = \overline{v}_1 \ldots \overline{v}_N \ , \qquad (88)$$

with all other components set to zero. We also see that the rank of M is at most N_c. If it is less than N_c, then B or $\overline{B}$ (or both) vanish. If the rank of M is k, then $SU(N_c)$ is broken to $SU(N_c - k)$ with $N_f - k$ massless flavors.

3.3 Supersymmetry is Unbroken in Pure Super Yang Mill Theory

Just see Kendrick Smith's paper[6]

3.4 Supersymmetric QCD(SQCD), Affleck-Dine-Seiberg Superpotential

Consider $SU(N_c)$ SUSY QCD with N_f flavors (that is there are $2N_cN_f$ chiral supermultiplets) where $N_f < N_c$ so that the theory is asymptotical free($b = 3N_c - N_f > 0$) and there are no baryons. We will denote the quarks and their superpartner squarks that transform in the $SU(N_c)$ fundamental (defining) representation by Q and Φ respectively, and use $\overline{Q}$ and $\overline{\Phi}$ for the quarks and squarks in the anti-fundamental representation. The theory has an $SU(N_f) \times SU(N_f) \times U(1) \times U(1)_R$ global symmetry. The quantum numbers of the chiral supermultiplets are summarized in the following table in more detain compared to Table. 70.

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)$	$U(1)_R$	$U(1)_A$	$U(1)_{R'}$
Φ, Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{N_f - N_c}{N_f}$	1	1
$\overline{\Phi}$, $\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{N_f - N_c}{N_f}$	1	1
Λ^b	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$2N_f$	$2N_c$
m^i_j	$\mathbf{1}$	$\overline{\square}$	$\square$	0	$\frac{2N_c}{N_f}$	-2	0

(89)

We assign the proper R charge to $\Phi, Q, \bar{\Phi}, \bar{Q}$ so that $U(1)$ is anomaly free. The axial $U(1)_A$ symmetry is an anomalous symmetry which is explicitly broken by instantons. To keep track of selection rules arising from the broken $U(1)_A$ we can define a spurious symmetry in the usual way. The transformations

$$\begin{aligned} Q &\rightarrow e^{i\alpha} Q \ , \\ \overline{Q} &\rightarrow e^{i\alpha} \overline{Q} \ , \\ \theta_{\mathrm{YM}} &\rightarrow \theta_{\mathrm{YM}} + 2N_f \alpha \ , \end{aligned} \tag{90}$$

leave the path integral invariant. Under this transformation the holomorphic intrinsic scale (61) transforms as

$$\Lambda^b \rightarrow e^{i2F\alpha} \Lambda^b \ . \tag{91}$$

By absorbing θ_{YM} into complex Λ and making it charged under $U(1)_A$, we can simply treat the anomalous symmetry $U(1)_A$ just as the anomalous free symmetry. We also introduce the R' symmetry which is a combination of R, A ($R' = R + N_c A/N_f$) so that the mass parameter m_i^j (the corresponding mass term is $m_j^i \overline{\Phi}^{jn} \Phi_{ni}$) carry no R' charge.

At tree level, in the absence of a superpotential, a supersymmetric gauge theory typically has a large set of vacua. These are the points with vanishing D-terms.

$$D_a \equiv -g \sum_{ij} \phi_i^* T_a^{ij} \phi_j = 0 \ . \tag{92}$$

Understanding the space of flat directions (usually referred to as classical moduli space) is crucial to study a model. It is often a non-trivial problem to find explicitly all the solutions to Eq. (92). In some cases the techniques of refs. [4, 5] can be useful. Fortunately there is a general theorem [16] stating that the space of solutions to Eq. (92) is in a *one-to-one correspondence* with the VEVs of the complete set of *complex* gauge-invariant functions of the chiral fields ϕ_i. In other words, the moduli space is the space of independent chiral invariants. The coordinates on the moduli space correspond to massless chiral supermultiplets. In general, the global description of this space is given in terms of a set of invariants satisfying certain constraints. So instead of trying to solve Eq. (92) explicitly, we turn to find all the invariants and the constraints which greatly simplifies the search for the solutions of eq. (92), and in practice it is very useful.

After adding a superpotential W, some flat directions are lifted, the F terms are non-vanishing along the D-flat direction. In particular, if one can show that every invariant is fixed by the condition $F_i = -\partial_{\phi_i} W = 0$, then *all* flat directions have been lifted, and the first requirement of the ADS criterion is satisfied. In terms of the invariants the vacua are described by the zeros of holomorphic functions, and this simplifies things considerably.

The (D-flat) classical moduli space (space of VEVs where the potential V vanishes) here is given by

$$\langle \overline{\Phi}^* \rangle = \langle \Phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix} \qquad (93)$$

where $\langle \Phi \rangle$ is a matrix with N rows and F columns and we have used global and gauge symmetries to rotate $\langle \Phi \rangle$ to a simple form. At a generic point in the moduli space the $SU(N_c)$ gauge symmetry is broken to $SU(N_c - N_f)$. There are

$$N_c^2 - 1 - ((N_c - N_f)^2 - 1) = 2N_c N_f - N_f^2 \qquad (94)$$

broken generators, so of the original $2N_c N_f$ chiral supermultiplets only N_f^2 singlets are left massless. This is because in the supersymmetric Higgs mechanism a massless vector supermultiplet "eats" an entire chiral supermultiplet to form a massive vector supermultiplet. We can describe the remaining N_f^2 light degrees of freedom in a gauge invariant way by an $N_f \times N_f$ matrix field

$$M_i^j = \overline{\Phi}^{jn} \Phi_{ni} \ , \qquad (95)$$

where we sum over the color index n. Note that M is an invariant quantity under complex gauge transformations, the issue of SUSY breaking in the theory could be described only by F term constraints $dW = 0$. Because of holomorphy, the only renormalization of M is the product of wavefunction renormalizations for Φ and $\overline{\Phi}$.

Note that $\det M$ is the only $SU(N_f) \times SU(N_f)$ invariant we can make out of M. To be invariant, a general non-perturbative term in the Wilsonian superpotential must have the form

$$\Lambda^{bn}(W^a W^a)^m (\det M)^p \ . \tag{96}$$

As usual to preserve the periodicity of θ_{YM} we can only have powers of Λ^b. Since the superpotential is neutral under $U(1)_A$ and has charge 2 under $U(1)_R$, the two symmetries require:

$$0 = n\,2N_f + p\,2N_f = 2m + p\,2(N_f - N_c) \ . \tag{97}$$

The solution of these equations is

$$n = -p = \frac{1-m}{N_c - N_f} \ . \tag{98}$$

Since $b = 3N_c - N_f > 0$ we can only have a sensible weak-coupling limit ($\Lambda \to 0$) if $n \geq 0$, which implies $p \leq 0$ and (because $N_c > N_f$) $m \leq 1$. Since $W^a W^a$ contains derivative terms, locality requires $m \geq 0$ and that m is integer valued. In other words, since we trying to find a Wilsonian effective action (which corresponds to performing the path integral over field modes with momenta larger than a scale μ) which is valid at low-energies (momenta below μ) it must have a sensible derivative expansion. So there are only two possible terms in the effective superpotential: $m = 0$ and $m = 1$. The $m = 1$ term is just the tree-level field strength term. We see that the gauge coupling receives no non-perturbative renormalizations. The other term ($m = 0$) is the Affleck-Dine-Seiberg superpotential:

$$W_{\rm ADS} = C_{N_c,N_f} \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}} \ , \tag{99}$$

where C_{N_c,N_f} is in general renormalization scheme dependent. Comparing to Eq.(66) and (69), we can find the Affleck-Dine-Seiberg superpotential is actually very similar to the instanton corrections to the superpotential at the tree level. We will mention it later that this is actually true when $N_f = N_c - 1$, where Eq.(66) is coincident to the one-instanton amplitude.

The superpotential $W_{\rm ADS}$ has no supersymmetric minima at finite M, but vanishes at infinity as an inverse power of M. This is easy to see that

$$\begin{aligned} V &= \sum_i |\frac{\partial W}{\partial Q_i}|^2 + |\frac{\partial W}{\partial \overline{Q}_i}|^2 \\ &= \sum_i |F_i|^2 + |\overline{F}_i|^2 \ , \end{aligned} \tag{100}$$

is minimized as $\det M \to \infty$, so there is a "run-away vacuum", or more strictly speaking no vacuum. It is usually assumed that this cannot happen unless there are particles that become massless at some point in the field space, which would also produce a singularity in the superpotential.

Since Λ runs only at one loop, its matching at thresholds at which heavy states are integrated out is simply done at tree level by requiring continuity through the threshold. For example, if we integrate out one flavour of quark superfields with mass m in SQCD, the effective scale of the low-energy $N_f - 1$ theory is simply

$$\Lambda_{eff}^{3N_c - N_f + 1} = m \ \Lambda^{3N_c - N_f} \ . \tag{101}$$

just like the normal QCD at one loop. In the previous case, by giving a VEV to just one meson $M_{N_f}^{N_f}$ in massless SQCD, the gauge group is broken down to $SU(N_c - 1)$ with $N_f - 1$ flavours, as one flavour disappears because it is eaten by the Higgs mechanism. In this case the low-energy scale is $\Lambda_{eff}^{3N_c - N_f - 2} = \Lambda^{3N_c - N_f} / M_{N_f}^{N_f}$. For $N_f < N_c$, when all mesons acquire VEVs, the low-energy theory is pure $SU(N_c - N_f)$ with a scale

$$\Lambda_{eff}^{3(N_c - N_f)} = \frac{\Lambda^{3N_c - N_f}}{\det M} \ . \tag{102}$$

The coefficients before the ADS superpotential could be obtained by instanton calculations. When $N_f = N_c - 1$, at a generic point $\det M \neq 0$, all flavours get VEVs, and the gauge group is completely broken. The ADS superpotential is simply

$$W_{eff} = C\ \frac{\Lambda^{2N_c+1}}{\det M}, \tag{103}$$

where c is a constant. Notice that $\Lambda^{2N_c+1} \propto \exp(-8\pi^2/g_W^2)$, which is precisely the suppression factor of a one-instanton amplitude. The calculation of instanton effects in the broken phase is reliable, as large instantons are suppressed by the gauge-boson mass, and this calculation explicitly shows that $c \neq 0$ [8]. When $N_f < N_c - 1$, unlike the case $N_f = N_c - 1$, the power of Λ_{N_c,N_f} does not coincide with the one-instanton effect, so we cannot perform a direct calculation of the constant C_{N_c,N_f}. Indeed, at a generic point on the classical moduli space, there is now an unbroken $SU(N_c - N_f)$ gauge group. So one expects additional non-perturbative effects other than instantons. However, we can obtain C_{N_c,N_f} from the theory with $N_c - 1$ flavours by adding a mass m to $N_c - 1 - N_f$ flavours. The exact superpotential for this theory is just eq. (101) plus the mass term [9]. By integrating out the mesons containing a massive quark, we obtain W_{eff} for the theory with N_f flavours. This has the form of Eq. (100) with $\Lambda_{N_c,N_f}^{3N_c-N_f} = m^{N_c-N_f-1}\Lambda^{2N_c+1}$, which is precisely the scale determined by matching the theory with N_f to the theory with $N_c - 1$ flavours, see Eq. (101). The constant C_{N_c,N_f} can then be computed in terms of the constant C, and one concludes that $c' \neq 0$ [10, 11]. By absorbing the order one parameter C or C_{N_c,N_f} into Λ, we can say the ADS superpotential is *exact* without the coefficients.

If we consider the extension to the massive case where $W_{tree} = \mathrm{Tr}\,(mM)$, and $\det(m) \neq 0$. Adding a mass term $\mathrm{Tr}\,(mM)$ to the general ADS superpotential Eq. (99), one finds N_c supersymmetric vacua characterized by

$$\langle M_i^j \rangle = (m^{-1})_i^j \left(\det m \Lambda^{3N_c-N_f}\right)^{\frac{1}{N_c}}, \tag{104}$$

corresponding to the N_c branches of the N_c-th root. Notice that N_c is precisely the number of vacua suggested by the Witten index Tr $(-1)^F = N_c$, calculated in supersymmetric pure $SU(N_c)$ theories at finite volume.

Recalling the expression of the dynamical scale for the effective theory, see Eq. (101), we observe that the superpotential in Eq. (99) is $W_{eff} = \Lambda^3_{eff}$. This is precisely the term that would be generated by the gauge kinetic term $\int d^2\theta W^\alpha W_\alpha$ in $SU(N_c - N_f)$, if the glueball field $W^\alpha W_\alpha$ were to receive a VEV $\sim \Lambda^3_{eff}$. Therefore the interpretation of E q. (99) is just that gauginos $\lambda^\alpha \lambda_\alpha = W^\alpha W_\alpha|_{\theta=\bar{\theta}=0}$ condense in the vacuum of the low-energy pure $SU(N_c - N_f)$ theory. This result confirms other approaches where $\langle\lambda\lambda\rangle = \Lambda^3_{eff}$ was derived by direct instanton calculus [12]. Notice that the $SU(N_c)$ theory has a discrete Z_{2N_c} R symmetry under which $\lambda \to e^{2\pi i k/2N_c}\lambda$, $k = 1, \ldots, 2N_c$, broken down to Z_2 by $\langle\lambda\lambda\rangle \neq 0$. Again, the resulting N_c-equivalent vacua are in agreement with the index $\mathrm{Tr}(-1)^F = N_c$. Notice that this is a very good example that shows gaugino condensation does not necessarily mean SUSY is dynamically broken.

3.5 The Quantum Moduli Space for $N_f = N_c$

For $N_f = N_c$, SQCD confines [13] with the light bound states given by the meson matrix M_i^j and the baryons B, $\bar{B}$. We will prove it in the Appendix that the quantum effects (instantons) will modify the classical constraint $\det M - B\bar{B} = 0$ to

$$\det M - B\bar{B} = \Lambda^{2N_c}. \tag{105}$$

This field equation, defining the so-called quantum moduli space (QMS), can be imposed by introducing a Lagrange-multiplier superfield A with superpotential

$$W_{quantum} = A\left(\det M - B\bar{B} - \Lambda^{2N_c}\right). \tag{106}$$

Notice that on any point of the QMS the field A pairs up with a linear combination of $M, B, \bar{B}$ and becomes massive. The above picture was derived inductively in ref. [13], as it satisfies a series of non-trivial consistency checks. In particular the massless spectrum from eq. (106) satisfies at any point 't Hooft's anomaly matching conditions, for flavour and R symmetries. Thus for $N_f = N_c$ massless SQCD not only exists but has an infinite degeneracy of vacua.

3.6 3-2 Model

In the mid 1980s, Affleck, Dine, and Seiberg [5] found the simplest known model with calculable dynamical SUSY breaking. The theory is QCD, with three colors and 2 flavors, $\overline{Q} = (\overline{U}, \overline{D})$. The authors gauged the SU(2) flavor symmetry acting on Q to mimic Weinherg-Salam model[10], and add a weak doublet L to cancel the SU(2) anomaly. The model has a gauge group $SU(3)\times SU(2)$ and two global $U(1)$ symmetries with the following chiral supermultiplets:

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$1/3$	1
L	$\mathbf{1}$	$\square$	-1	-3
$\overline{U}$	$\overline{\square}$	$\mathbf{1}$	$-4/3$	-8
$\overline{D}$	$\overline{\square}$	$\mathbf{1}$	$2/3$	4

(107)

[10]This would be the Weinherg-Salam model with one generation of quarks and leptons if we added the positron e^+, and gauged hypercharge.

I denote the intrinsic scales of the two gauge groups by Λ_3 and Λ_2 respectively. With no superpotential, this theory has flat directions:

$$\langle\overline{Q}\rangle = \langle Q\rangle = \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 0, & \sqrt{a^2-b^2} \end{pmatrix} \tag{108}$$

A complete set of gauge invariants is

$$X = Q\bar{D}L, \qquad Y = Q\bar{U}L, \qquad Z = \det\bar{Q}Q = Q\bar{U}Q\bar{D}. \tag{109}$$

For $\Lambda_3 \gg \Lambda_2$ (that is we neglect the non-perturbative effects for the $SU(2)$ gauge interaction), instantons give the standard Affleck-Dine-Seiberg superpotential:

$$W_{\rm dyn} = \frac{\Lambda_3^7}{\det(\overline{Q}Q)} \ , \tag{110}$$

which has a runaway vacuum. Adding a tree-level trilinear term to the superpotential

$$W = \frac{\Lambda_3^7}{\det(\overline{Q}Q)} + \lambda\, Q\bar{D}L \ , \tag{111}$$

removes the classical flat directions and produces a stable minimum for the potential. Since the vacuum is driven away from the point where the VEVs vanish by the dynamical ADS potential (110), the global $U(1)_R$ symmetries are broken and we expect (by the rule of thumb described above) that SUSY is broken.

The L equation of motion

$$\frac{\partial W}{\partial L_\alpha} = \lambda \epsilon^{\alpha\beta} Q_{m\alpha}\overline{D}^m = 0 \ , \tag{112}$$

tries to set $\det\overline{Q}Q$ to zero since

$$\det\overline{Q}Q \quad = \quad \det \begin{pmatrix} \overline{U}Q_1 & \overline{U}Q_2 \\ \overline{D}Q_1 & \overline{D}Q_2 \end{pmatrix}$$

$$= \overline{U}^m Q_{m\alpha} \overline{D}^n Q_{n\beta} \epsilon^{\alpha\beta} \ . \tag{113}$$

In the similar way, multiplying the equation $0 = \partial_{\bar{U}} W$ by $\bar{U}$ and $\bar{D}$, we get $X = 0$ and $Y = 0$, respectively. This procedure determine all independent chiral invariants X, Y, and Z, then all D-flat directions have been lifted by the superpotential. However, notice that the potential here can't have a zero-energy minimum since the dynamical term blows up at $\det\overline{Q}Q$=0. Therefore SUSY is indeed broken.

We can crudely estimate the vacuum energy for by taking all the VEVs to be of order ϕ. For $\phi \gg \Lambda_3$ and $\lambda \ll 1$ we are in a perturbative regime. The potential is then given by

$$V = |\frac{\partial W}{\partial Q}|^2 + |\frac{\partial W}{\partial \overline{U}}|^2 + |\frac{\partial W}{\partial \overline{D}}|^2 + |\frac{\partial W}{\partial L}|^2 \tag{114}$$

$$\approx \frac{\Lambda_3^{14}}{\phi^{10}} + \lambda \frac{\Lambda_3^7}{\phi^3} + \lambda^2 \phi^4 \ , \tag{115}$$

where in the last line we have dropped the numerical factors since we are only interested in the scaling behavior of the solution. This potential has a minimum near

$$\langle \phi \rangle \approx \frac{\Lambda_3}{\lambda^{\frac{1}{7}}} \ , \tag{116}$$

Plugging the solution back into the potential (113) we find the vacuum energy is of order

$$V \approx \lambda^{\frac{10}{7}} \Lambda_3^4 \ , \tag{117}$$

which vanishes as λ or Λ go to zero. When the theory is weakly coupled for small λ as $\phi \gg \Lambda_3$. Non-perturbative corrections to the Kähler potential are negligible and typically the tree-level approximation is sufficient to characterize the spectrum around the minimum of the potential. Indeed, in the limit $\lambda^{1/7} \ll 1$, the three original moduli X, Y, Z describe the light degrees of freedom. The vacuum can be studied by considering the non-linear σ-model for the light states X, Y, Z, obtained by integrating out the heavy modes in a theory with an originally flat tree-level Kähler metric [5].

Using duality Intriligator and Thomas [14] showed that we can also understand the case where $\Lambda_2 \gg \Lambda_3$ and supersymmetry is broken non-perturbatively. The $SU(2)$ gauge group has 4 doublets which is equivalent to 2 flavors, so we have confinement with chiral symmetry breaking. The $SU(3)$ gauge group has two flavors and is completely broken for generic VEVs. It is simpler to consider $SU(2)$ as an SU group rather than an Sp group, so we write the gauge invariant composites as mesons and baryons:

$$\begin{aligned} M &\sim \begin{pmatrix} LQ_1 & LQ_2 \\ Q_3Q_1 & Q_3Q_2 \end{pmatrix} , \\ B &\sim Q_1Q_2 , \\ \bar{B} &\sim Q_3L . \end{aligned} \tag{118}$$

In this notation the effective superpotential is

$$\begin{aligned} W &= X \left(\det M - B\overline{B} - \Lambda_2^4\right) + \lambda \sum_{i=1}^{3} Q_i \bar{D}_i L \\ &= X \left(\det M - B\overline{B} - \Lambda_2^4\right) + \lambda \left(\sum_{i=1}^{2} M_{1\mathrm{i}} \overline{D}^{\mathrm{i}} + \overline{B}\,\overline{D}^3 \right) , \end{aligned} \tag{119}$$

where X is a Lagrange multiplier field that imposes the constraint for confinement with chiral symmetry breaking. The $\overline{D}$ equations of motion try to force M_{1i} and $\overline{B}$ to zero while the constraint means that at least one of M_{11}, M_{12}, or $\overline{B}$ is non-zero, so we see that SUSY is broken at tree-level in the dual (confined) description. We can estimate the vacuum energy as

$$V \approx \lambda^2 \Lambda_2^4 . \tag{120}$$

Comparing the vacuum energies in the two cases we see that the $SU(3)$ interactions dominate when $\Lambda_3 \gg \lambda^{\frac{1}{7}}\Lambda_2$.

Without making the approximation that one gauge group is much stronger than the other we should consider the full superpotential

$$W = A\left(\mathrm{det}M - B\bar{B} - \Lambda_2^4\right) + \frac{\Lambda_3^7}{\mathrm{det}(\overline{Q}Q)} + \lambda\, Q\bar{D}L\ , \tag{121}$$

which still breaks SUSY, although the analysis is more complicated.

3.7 Dynamically SUSY breaking and Quantum Deformed Moduli Space

The Intriligator-Thomas-Izawa-Yanagida [15, 17] model is a vector-like theory which consists of an $SU(2)$ SUSY gauge theory with two flavors[11] and a gauge singlet:

	$SU(2)$	$SU(4)$	$U(1)_R$
Q	$\square$	$\square$	0
S	**1**	????	2

(122)

with the only renormalizable superpotential with the right symmetries

$$W = \lambda S^{ij} Q_i Q_j\ . \tag{123}$$

The strong $SU(2)$ dynamics enforces a constraint

$$\mathrm{Pf}(M) = \Lambda^4\ . \tag{124}$$

[11]Since doublets and anti-doublets of $SU(2)$ are equivalent, an $SU(2)$ theory with N_f flavors has a global $SU(2N_f)$ symmetry rather than an $SU(N_f)\times SU(N_f)$ as one finds for a larger number of colors.

Where the Pfaffian[12] of a $2N_f \times 2N_f$ matrix M is given by

$$\mathrm{Pf}(M) = \epsilon^{i_1 \ldots i_{2F}} M_{i_1 i_2} \ldots M_{i_{2F-1} i_{2F}} \ . \tag{125}$$

The equation of motion for for the gauge singlet S is

$$\frac{\partial W}{\partial S^{ij}} = \lambda Q_i Q_j = 0 \ . \tag{126}$$

Since this equation is incompatible with the constraint (124) we see that SUSY is broken.

Another way to see this is that at least for large values of λS we can integrate out the quarks, leaving an $SU(2)$ gauge theory with no flavors which has gaugino condensation:

$$\Lambda_{\rm eff}^{3N} = \Lambda^{3N-2} \left(\lambda S\right)^2 \ , \tag{127}$$

$$W_{\rm eff} = 2\Lambda_{\rm eff}^3 = 2\Lambda^2 \lambda S \ , \tag{128}$$

$$\frac{\partial W_{\rm eff}}{\partial S^{ij}} = 2\lambda\Lambda^2 \ , \tag{129}$$

so again we see that the vacuum energy is non-zero.

For general values of λS we can write:

$$W_{\rm eff} = \lambda S^{ij} Q_i Q_j + X(\mathrm{Pf}M - \Lambda^4) \ , \tag{130}$$

where X is a Lagrange multiplier field. For $\lambda \ll 1$ the vacuum is close to the SUSY QCD vacuum given by the X equation of motion, and we can treat the first term in the superpotential as a small mass perturbation.

The potential energy is given by:

$$V = \sum_i |\frac{\partial W_{\rm eff}}{\partial Q_i}|^2 + \sum_{ij} |\frac{\partial W_{\rm eff}}{\partial S^{ij}}|^2 \ . \tag{131}$$

[12] If we artificially divided Q up into q and $\overline{q}$ then we could follow our previous notation $M = q\overline{q}$, $B = \epsilon^{ij} q_i q_j$, $\overline{B} = \epsilon^{ij} \overline{q}_i \overline{q}_j$ and write the constraint as $\mathrm{Pf}(QQ) = \det M - B\overline{B} = \Lambda^4$.

A supersymmetric vacuum exists if all the terms vanish. Treating λS as a mass perturbation we can set the derivatives with respect to Q to zero simply by solving for the squark VEVs in the standard way. This gives

$$Q_i Q_j = \left(\mathrm{Pf}(\lambda S)\Lambda^{3N-F}\right)^{\frac{1}{N}} \left(\frac{1}{\lambda S}\right)_{ij} \quad . \tag{132}$$

Plugging this back in to the potential gives

$$\begin{aligned} V &= \sum_{ij} \left|\frac{\partial W_{\mathrm{eff}}}{\partial S^{ij}}\right|^2 = |\lambda|^2 \sum_{ij} |M_{ij}|^2 \ , \\ &= |\lambda|^2 |\mathrm{Pf} S \Lambda^4| \sum_{ij} \left| \left(\frac{1}{S}\right)_{ij} \right. \ , \end{aligned} \tag{133}$$

which is minimized at

$$S^{ij} = (\mathrm{Pf}S)^{\frac{1}{2}} \epsilon^{ij} \tag{134}$$

so

$$V = 4|\lambda|^2 \Lambda^4 \tag{135}$$

which agrees with the previous result from gaugino condensation and 3-2 model in the limit of $\Lambda_2 \gg \Lambda_3$.

Since this theory is vector-like (it admits mass terms for all the quarks and for S) one would naively expect that this model could not break SUSY. This is because the Witten index $\mathrm{Tr}(-1)^F$ is non-zero with mass terms turned on so there is at least one supersymmetric vacuum. Since the index is topological, it does not change under variations of the mass. However Witten noted that there is a loop-hole in the index argument since the potential for large field values are very different with $\Delta W = m_s S^2$ from the theory with $m_s \to 0$, since in this limit vacua can come in from or go out to ∞ and thus change the index.

4 Outlook and Comments

After I have finished studying SUSY phenomenology, non-perturbative SUSY gauge theory and DSB. We will ask how DSB is connected to the TeV SUSY phenomenology?

The answer is that DSB or more generally, SUSY breaking is connected to the TeV SUSY phenomenology, the soft SUSY breaking terms indirectly. The SUSY breaking happens in the hidden sector can not direct communicate with the visible sector(what we will observe in LHC). There is an additional messager sector that connects them. Due to different messagers, we classify them as two different scenarios: Gravity mediated SUSY breaking, whereas the meesager is gravity; Gauge mediated SUSY breaking, whereas the messager is particles with gauge symmetry. The bulk mediated SUSY breaking, which has extra dimensions in the theory, could be also divided into anomaly-mediated SUSY breaking(the messager is gravity) and gaugino mediated SUSY breaking(the messager is particles with gauge symmetry).

For all the realistic SUSY model building, there are three major aspects we must be careful: They are

- Tree level mass sum rule. The spontaneous breaking of supersymmetry at tree level leads to a mass relation

  $$STrM^2 = \sum_J (-1)^{2J}(2J+1)M_J^2 = \sum \frac{1}{2} g_\alpha^2 \langle D^\alpha \rangle Tr(t^\alpha) \ , \tag{136}$$

  where m_J is the mass matrix for spin J field, α runs only over U(1) factors. If all the U(1) symmetry is anomaly free, then $Tr(t^\alpha) = 0$ and $STrM^2$ is zero. This is troublesome since in realistic model buildings, we need more scalar masses than fermion masses.

- Precise electroweak measurements. Mostly they are associated to how to break eletroweak symmetry to mimic the Higgs mechanism?

- Low energy constraints from the rare decay process and electric dipole moment(EDM). Mostly they are associated with the flavor problems in the Standard Model. Especially fobiding the dangerous flavor changing neutral current(FCNC) and large CP violating interaction at low energy.

The first problem, such tree level mass sum rule could be violated in two different aspects:

- Supertrace tree level sum rule is violated due to the fact that SUSY is a local symmetry. For n chiral superfields, minimal kinetic terms, and a vanishing bosonic potential, the supertrace formula becomes:

$$STrM^2 = (n-1)\Big(m_{3/2}^2 - D^a D^a / M^2\Big). \tag{137}$$

  With vanishing D-terms, we see that the average scalar mass must exceed that of its chiral fermion partner by an amount $m_{3/2}$, which is exactly what we want. (For a nice discussion, see David Morrissey's notes on SUSY)[23]

- Supertrace sum rule, however, does not hold beyond tree level. It was soon realized that when the splittings inside supermultiplets arise from radiative corrections, the sparticles can all be made consistently heavier than the SM particles. This was a motivation of the first gauge-mediated supersymmetry-breaking models. So it leads to the dynamical SUSY breaking in the hidden sector and such mass gap could be transmitted to the visible sector without supergravity.

Because we can not make a TeV supergravity model. So building a good visible SUSY breaking model perhaps suggests that such visible model should be a Visible Dynamical Supersymmetry Breaking(VDSB). However, this is the motivation of the first gauge-mediated supersymmetry-breaking models. Indeed the aim of refs. [24, 25, 26, 27] was to build a "supersymmetric technicolour" theory, in which the breakdown of supersymmetry is due to some strong gauge dynamics, while its mediation to the SM particles is just due to the usual SM gauge interactions. However, all the Higgsless model suffers a lot from the precise electroweak measurements simply because it is so hard to mimic the Higgs mechanism without a fundamental Higgs. Standard Model itself is so successful and looks really mysterious to me! So I would still hold the fact that Higgs boson is elementary. However, the motivation for a strong dynamics in TeV is totally different in a supersymmetric theory from a non-supersymmetric theory. Just as I have divided the GHP in two parts in the previous section. In a supersymmetric theory, the strong dynamics is only necessary to generate the big susy breaking scale from the fundamental scale, while in a nonsupersymmetric theory, in the absence of powerful symmetry which kills the quadratic divergence, the strong dynamics is laos responsible to the Higgs mechanism, which makes it hard to survive from the precise electroweak measurements.

So, in a generic aspect, I would think about VDSB in the following way:

- Try to find in what conditions, the non-perturbative effects would give us the right magnitude of the mass gap between scalars and fermions just like in SUGRA.

- Strong dynamics is not associated with the Higgs sector, perhaps with some additional singlets in the NMSSM, which has been suggested by Harvard group[28]. The strong dynamics just exist in the small energy near the TeV region so we still have SUSY GUT.

- The flavor problems are perhaps very troublesome for the visible SUSY breaking since it is hard to impose any additional nice condition like the universality condition for the gravity-mediated SUSY breaking or flavor-blinding in gauge-mediated SUSY breaking.(I am still not very familiar with flavor problems.)

Finally, we may ask what have we gained if a strongly-coupled SUSY theory(SUSY is dynamically broken) exist in the TeV?

- It will push the Higgs mass. The new lower bounds for the light CP even Higgs mass from LEP II is higher than the perturbative upper bounds we get above. So fine-tunning of the stop mass is needed in MSSM.
- Running into new territory of SUSY parameter, especially Large yukawa couplings: The parameters spaces with the strong dynamics should be different. Just from Section 2.5, another strong gauge couplings will require a new large yukawa coupling to balance their contribution in the RG equations.
- Large yukawa couplings will be an interesting solution to dark matter, EW baryogenesis. See Ref. [29].
- New moduli cosmology, since there are many D-flat moduli spaces so the properties of light or massless scalar fields in TeV are different.

Appendix: Quantum Deformation of the Classical Constraints on the Moduli Space $N_c = N_f$

The classical moduli space for $N_c = 2$ are

$$\langle \Phi \rangle = \begin{pmatrix} a & \\ & a \end{pmatrix} \tag{A.138}$$

They can be described by the gauge invariant combinations

$$V^{ij} = Q^i Q^j \ . \tag{A.139}$$

Note here V is just a combination of M and B as $\mathbf{2} = \bar{\mathbf{2}}$ in the $SU(2)$. The classical moduli space constraint can be described as

$$\mathrm{Pf}V \equiv \epsilon_{i_1,\ldots,i_{2N_f}} V^{i_1 i_2} V^{i_3 i_4} = 0 \ . \tag{A.140}$$

(which is meaningful only for $N_f \geq 2$) Adding a superpotential mass term $W_{\text{tree}} = \mathrm{Tr}(m_{ij} V^{ij})$, the symmetries, the weak couling and small mass limit will determine that the exact superpotential is $W_{full} = W_{ADS} + W_{tree}$[13]. The gives a solution for V:

$$V^{ij} = \langle Q^i Q^j \rangle \sim \Lambda^{\frac{6-N_f}{2}} (\mathrm{Pf}\ m)^{1/2} \left(\frac{1}{m}\right)^{ij} \ . \tag{A.141}$$

Since the masses are holomorphic parameters of the theory, this relationship can only break down at isolated singular points, so Eq. (104), (A.141) is true for generic masses and VEVs. Explicitly calculations[5, 8] show that the coefficients of order one in these relations do not vanish. Therefore, we can redefine Λ such that

$$V^{ij} = \langle Q^i Q^j \rangle = \Lambda^{\frac{6-N_f}{2}} (\mathrm{Pf}\ m)^{1/2} \left(\frac{1}{m}\right)^{ij} \ . \tag{A.142}$$

Then it is easy to see that the classical constraints Eq.(A.140) is modified quantum mechanically to

$$\mathrm{Pf}V = \Lambda^4 \qquad \text{for} N_c = 2 \ , \tag{A.143}$$

which is *independent* of m, for $N_f = N_c$. The result can be also extended to the case without mass term by taking the limit $m \to 0$.

[13]Although ADS superpotential made no sense for $N_f \geq N_c$ however the vacuum solution Eq. (104) is still sensible.

In a similar way, we have Eq. (104)

$$M^i_j = \langle Q^i \tilde{Q}_j \rangle = \Lambda^{\frac{3N_c - N_f}{N_c}} (\det\, m)^{1/N_c} \left(\frac{1}{m}\right)^i_j . \tag{A.144}$$

This agrees with the constraints Eq. (105) for $B = \tilde{B} = 0$. The more general case with $W_{tree} = \mathrm{Tr} MN + bB + \tilde{b}\tilde{B}$ with $m, b, \tilde{B} \to 0$, $\langle B\bar{B} \rangle$ can be non-zero and expectation values are found to satisfy Eq. (105)[22].

References

[1] In principle, we could add as many singlets N_i as we want to MSSM since it does not affect the anomaly cancellation. The D-term potential will be zero. If we choose a proper superpotential $W(N_i, \cdots)$ so that the F-term for the scalar component is still in the flat direction, we would call such scalar component moduli, which means gauge inequivalent vacuum states. The scalar component for the Φ_1(The superfield appear in the linear term of the superpotential) in O'Raifeartaigh SUSY breaking gives a perfect example. Such moduli fields has many non-trivial properties in cosmology.

[2] The reason that we do not use B and L symmetry in MSSM is that they are just additional symmetry which is violated by non-perturbative electroweak effects, and they are believed violated at very high energy scale which is essential to generate a baryon asymmetric universe. The reason that we do not use $U(1)_{B-L}$ is that we also want to incorporate the Majarana mass terms which would violate fermion number by 2 if they are in the chiral supermutiplets(gaugino has no lepton though they are Majarana particle). If we believe the tiny light neutrino masses are generated due to seesaw mechanism, then we need to introduce a majarana mass term for the heavy right-handed neutrinos, which will violate $B-L$ but not R parity. Actually, there are still Dirac seesaw mechanism in the framework of anomaly-mediated SUSY breaking with $U(1)_{B-L}$ symmetry, for a brief review, see Ref.[3]

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