Supersymmetric Cosmic Strings

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Abstract

We study the formation of cosmic strings in the context of spontaneous symmetry breaking of gauge symmetry groups. We discuss the conditions for formation and stability of defects in nonsupersymmetric theories and also F-strings in the supersymmetry context. Finally, we discuss how cosmological observations can constrain supersymmetric grand unified theories.

1 Introduction

In the modern view of cosmology, the universe underwent a series of phase transitions as it expanded and cooled down after the big bang. As a result of these transitions, most models of grand unified theories (GUT) predict the existence of topological defects. These defects have a large energy density and would be expected to have impact on the structure formation of the universe, gravitational lensing, CMB anisotropies, gravitational waves and cosmic rays, if they are formed after or at the late stages of inflation.

On the other hand, inflationary models solve many problems of the standard model of cosmology and explain structure formation adn CMB anisotropies. If we believe that inflationary models are true then there is not much room left for topological defects. Since most GUT models predict the existence of these defects, we should be able to constrain these GUT models by allowing only those that produce defects that are consistent with the inflationary picture. For instance, domain walls, because they are heavy, and monopoles, because they are too abundant, would have a great gravitational effect, which is inconsistent with current observations. Therefore, we have to either rule out GUT models that predict the existence of monopoles and domain walls or ensure that these defects are formed before or in the early stages of inflation. In the latter case, the exponential expansion of inflation dilutes the defects' energy density and they are not observed today.

Cosmic strings are also predicted by most GUT models and, differently from domain walls and monopoles, are not inconsistent with the inflationary picture. They can actually help us understand aspects of cosmology for which we have no explanation in view of the standard inflationary scenario.

It is believed that supersymmetry (SUSY) is a general symmetry of nature, connecting fermions and bosons. But if this is true, SUSY must be broken since we do not observe super-partners for all the particles in the standard model of particle physics. Studying SUSY GUT theories in the context of inflation and the defects produced by these theories may tell us then about constraints in the physics at high energies, by constraining GUT models, and also about cosmology.

This paper is organized in the following way. In section (2), we give an overview of topological defects and their formation. In section (3), we provide examples of formation of domain walls, monopoles and strings and discuss the conditions for the formation and topological stability of the defects in the context of quantum field theories. In section (4), we discuss cosmic strings in light of SUSY models, specializing to the F-theory. In section (5) the cosmological constraints of those models are analyzed and we conclude in section (6).

2 Topological defects overview

During its first seconds, the universe had a series of phase transitions, which might have left important observational signatures, including topological defects [9]:

- **GUT transition** : $E \sim 10^{15}$ GeV. Bellow this energy, the gauge group of particle physics degenerates from the grand unified G to the usual standard model $SU(3) \otimes SU(2) \otimes U(1)$ by the Higgs mechanism.

- **Electroweak transition**: $E \simeq 300$ GeV. Bellow this energy, the Higgs mechanism breaks the $SU(2) \otimes U(1)$ part of the standard model to yield distinct electromagnetic and weak interactions.

- **Quark-hadron transition**: $E \simeq 0.2$ GeV. This transition is related to quark confinement.

During these phase transitions, it is thought that defects are formed by the Kibble mechanism [7]. Suppose we have a N-component real scalar field ϕ, with a Lagrangian invariant under $O(N)$ and coupled to $N(N-1)/2$ vector fields A_μ

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V_{eff}(\phi) \tag{1}$$

where

$$
\begin{aligned}
D_\mu \phi &= \partial_\mu \phi - e A_\mu \phi & (2)\\
F^{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + e[A_\mu, A_\nu] & (3)
\end{aligned}
$$

At zero temperature, the effective potential is simply

$$V_{eff}(\phi) = V(\phi) = \frac{1}{2} g^2 (\phi^2 - \eta^2)^2 \tag{4}$$

In this case the $O(N)$ symmetry is spontaneously broken to $O(N-1)$, and ϕ acquires a vacuum expectation value η, i.e. in tree approximation

$$< \phi >^2 = \eta^2 \tag{5}$$

Notice that there is a manifold M of degenerate vacua solutions, in this case a circle of radius η. At a finite temperature T, the leading temperature dependence of the effective potential at high T and small coupling constant is calculated from one-loop diagrams [14]

$$V_{eff}(\phi) = \frac{1}{2} g^2 (\phi^2 - \eta^2)^2 + \frac{1}{48}[(N+2)g^2 + 6(N-1)e^2] T^2 \phi^2 \tag{6}$$

In this case, the minimum occurs at $\phi = 0$, and the symmetry is unbroken for T larger than the critical temperature

$$T_c = \eta \left(\frac{N+2}{12} + \frac{N-1}{2} \frac{e^2}{g^2} \right)^{-1/2} \tag{7}$$

Thus, the Higgs field ϕ starts at high temperatures with a zero expectation value and when the symmetry breaks bellow T_c, it acquires an expectation value rolling down to one of the possible vacua. Since there are no correlations on distances greater than the horizon scale, different points in space will choose different vacua in the manifold M. As these regions of different vacua come into causal contact, topological defects might be formed. The conditions for topological defects formation are topological constraints of the manifold M (see section (3)). Defect formation depends then on the structure of the manifold M, and ultimately on the symmetry that is being broken. In our $O(N)$ case in 3 spatial dimensions, different values of N will lead to different defects, namely:

- ($N = 1$) **Domain walls**: Two-dimensional defects dividing space into connected regions which are in one of the two possible phases.
- ($N = 2$) **Strings**: Linear defects, where the phase of ϕ changes by 2π in making one loop around the string.
- ($N = 3$) **Monopoles**: Point defects where the field ϕ points radially away from the defect.
- ($N = 4$) **Textures**: Here it is easier to think of the case of only 2 spatial dimensions. Then textures correspond to field configurations in which ϕ makes a rotation of π in its 3-dimensional internal space in going from $r = 0$ to $r = \infty$.

3 Defect formation and stability

The precise condition on defect formation rests on the topological structure of the manifold of degenerate vacua M. Let us see some examples before we define these conditions.
Example 1 - Domain Walls

Suppose we have a real scalar field ϕ with a Lagrangian invariant under the reflection $\phi \to -\phi$

$$\mathcal{L} = \frac{1}{2}(\nabla\phi)^2 - V(\phi) \tag{8}$$

Here the zero temperature potential is again $V(\phi) = \frac{1}{2}g^2(\phi^2 - \eta^2)^2$. The equilibrium states are $\phi = \phi_\pm = \pm\eta$. Therefore the manifold of degenerate vacua M is a disconnected set of two points in this case. When different regions come into contact, the set of points in which the field vanishes form a two-dimensional surface that separates the two phases and constitute the domain wall.
Example 2 - Monopoles

Suppose we have a scalar field with internal space ϕ^i, $i = 1, 2, 3$. The gauge group to be broken is $SO(3)$ and there are three gauge fields A^i_μ, $i = 1, 2, 3$. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}D^\mu\phi^i D_\mu\phi^i - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - V(\phi) \tag{9}$$

where the covariant derivatives and tensors are

$$D_\mu\phi^i = \partial_\mu\phi^i - e\epsilon^{ijk}A^j_\mu\phi^k \tag{10}$$
$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - e\epsilon^{ijk}A^j_\mu A^k_\nu \tag{11}$$

Suppose the potential $V(\phi)$ is arranged to give symmetry breaking with $|\phi| = v$, along some arbitrary direction, which at a given point we can choose to be the 3-axis of the scalar field's internal space. After some time, the scalar field will align itself in such a way that at all points of space the fields will all be parallel in their internal space. There will exist then an internal $U(1)$ symmetry of rotations about the Higgs axis that remained unbroken, and the corresponding gauge field A^3_μ would be the electromagnetic 4-potential. This model is not realistic, but it provides an example of a model that yields the $U(1)$ of electromagnetism after symmetry breaking. We shall see now that we actually have a monopole solution. It is important the definition of the covariant derivative here, because the extra piece in it provides a cancellation of terms that otherwise would produce a divergent energy. Consider now the time-independent solution

$$\phi^i = v\frac{r_i}{r} \tag{12}$$

$$A^j_i = \epsilon_{ijk}\frac{r_k}{er^2} \tag{13}$$

where r denotes real radial position and r_i the internal coordinate of the scalar field. This solution is clearly a topological defect in the sense that there is a winding of the Higgs as one performs circles about the origin. Now, at large radii, the Higgs field does become uniform , and we suppose it becomes uniform in the direction of third axis. The magnetic field associated is then given by $B_i = \frac{1}{2}\epsilon ijkF^3_{jk}$. Inserting the solution for A^j_i we obtain $B^i = \frac{r_i}{er^3}$ [9].
Example 3 - Strings
a) Global

Suppose we have a complex scalar field $\phi(x)$ and a Lagrangian with a *global* $U(1)$ symmetry $\phi \to \phi e^{i\alpha}$, with α constant

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - V(\phi) \tag{14}$$

$$V(\phi) = \frac{1}{2}\lambda\left(|\phi|^2 - \frac{1}{2}\eta^2\right)^2 \tag{15}$$

The equation of motion can be obtained from the Lagrangian Eq.(14)

$$\partial^2\phi + \lambda\phi(|\phi|^2 - \frac{1}{2}\eta^2) = 0 \tag{16}$$

The vacuum solution (ground state) is the one that solves the equation of motion and also minimizes the scalar potential $V(\phi)$. In this case it is $\phi = (\eta/\sqrt{2})e^{i\alpha}$, with α constant, because this solution sets $V = 0$, which is its minimum value. This solution (unlike the $\phi = 0$ solution) is not invariant under $U(1)$, and we say that the symmetry is spontaneously broken by the vacuum. The mass m_s of the scalar particle in the symmetry-breaking vacuum is $m_s^2 = \lambda\eta^2$, and we also have a massless Nambu-Goldstone boson associated with the broken global symmetry.

Besides the vacuum, there are also static solutions with non-zero energy density. Making the cylindrically symmetric ansatz

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s r) e^{in\varphi} \tag{17}$$

with (r, φ, z) cylindrical coordinates and n integer, leads to the field equations in terms of f

$$f'' + \frac{1}{\zeta} f' - \frac{n^2}{\zeta^2} f - \frac{1}{2}(f^2 - 1) f = 0 \tag{18}$$

where $\zeta = m_s r$ and we have the boundary conditions

$$f(0) = 0 \tag{19}$$
$$f(\infty) = 1 \tag{20}$$

The solution goes like $f \sim 1 - n^2/\zeta^2$, and since the energy density for this static ($\partial_t \phi = 0$) solution is

$$E = |\dot{\phi}|^2 + |\nabla\phi|^2 + V(\phi) \tag{21}$$

we see that E is well localized near the origin, because it has a r^{-2} tail at large r which comes from the angular part of the gradient term. This means the energy density of this solution is infinite. These solutions are known as global strings and in particle cosmology, they are usually associated with a spontaneously broken axial $U(1)$ symmetry.

b) Gauge

Suppose now that the $U(1)$ symmetry is a *local* (gauge) symmetry. This requires us to introduce the vector field A_μ and consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) \tag{22}$$

with

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \tag{23}$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{24}$$

The $U(1)$ local symmetry requires the vector field A_μ to transform too

$$\phi \rightarrow \phi e^{i\Lambda(x)} \tag{25}$$
$$A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu \Lambda(x) \tag{26}$$

where here $\Lambda(x)$ is a function. The field equations are now

$$D^2\phi + \lambda\phi(|\phi|^2 - 12\eta^2) = 0 \tag{27}$$
$$\partial_\nu F^{\mu\nu} + ie(\phi^\star D^\mu \phi - D^\mu \phi^\star \phi) = 0 \tag{28}$$

Here the Higgs still has mass $m_s^2 = \lambda\eta^2$, but now the Nambu-Goldstone boson is incorporated into the vector field, which gains a mass $m_v = e\eta$.

The cylindrically symmetric solution still exists, but its energy per unit length will be finite because of the covariante derivative. In the radial gauge $A_r = 0$, we write the ansatz

$$\phi = \frac{\eta}{\sqrt{2}} f(m_v r) e^{in\varphi} \tag{29}$$
$$A^i = \frac{n}{er}\varphi^i a(m_v r) \tag{30}$$

The asymptotic behaviour solutions are as $\zeta \rightarrow 0$

$$f \simeq f_0 \zeta^{|n|} \tag{31}$$
$$a \simeq a_0\zeta^2 - \frac{|n| f_0^2}{4(|n|+1)}\zeta^{2|n|+2} \tag{32}$$

and as $\zeta \rightarrow \infty$

$$f \simeq 1 - f_1 \zeta^{-1/2} e^{-\sqrt{\beta}\zeta} \tag{33}$$
$$a \simeq 1 - a_1 \zeta^{1/2} e^{-\zeta} \tag{34}$$

where $\zeta = m_v r$ and $\beta = \lambda/e^2 = (m_s/m_v)^2$. This local string has a tube of quantized magnetic flux

$$\int d^2x\mathbf{B}.\mathbf{z} = \int_{S^1_\infty} dx^i A^i = \frac{2\pi n}{e} \tag{35}$$

where S^1_∞ is a circle of infinite radius centred on the string. The energy per unit lenght of this string is $\mu = \int r dr d\varphi E(r)$ and it may calculated from Eq.(21), producing the finite quantity

$$\mu = \pi\eta^2\epsilon(\beta) \tag{36}$$

The condition for the existence of defects as well as their stability lies on the structure of the manifold of degenerate vacua M [7, 4]. In a cosmological context, we would start with the GUT gauge group and after a finite number of transitions we should end up at the standard model gauge group $SU(3) \otimes SU(2) \otimes U(1)$. Let us suppose that we start with a symmetry group G under which our theory is invariant, and after a first transition we are left with a subgroup H of G, and after a second transition we end at a subgroup K of H

$$G \to H \to K \tag{37}$$

After the first transition, we end up in one point of the manifold of degenerate vacua M. It turns out that the manifold of degenerate vacua M is actually the left coset G/H[7],[4]. To see this, assume $\phi_0 \in M$. Then if $g \in G$, $g\phi_0 \in M$. Now H is the new symmetry group after the transition, so we have for $h \in H$ that $h\phi_0 = \phi_0$. Therefore, $g\phi_0 = g^{'}\phi_0 \in M$ if and only if $g^{'} = gh$ with $h \in H$, and we see that there is actually a one-to-one correspondence between points in M and the left cosets of H in G.

In order to see if topological defects form during the first phase transition and are stable down to the second phase transition we have to compute the homotopy groups $\pi_k(M)$ of the manifold M. We can write then $M = G/H$. Defects of a order k exist if the k^{th} homotopy group is non-trivial ($\neq 1$)[7, 4, 5, 3]. These conditions can also be translated geometrically by the fact that a non-trivial homotopy group of order k implies the existence of non-contractible k-spheres in the manifold M. If $\pi_0(M) = \pi_0(G/H) \neq 1$, M is disconnected and we have the formation of domain walls. If $\pi_1(M) = \pi_1(G/H) \neq 1$, M has noncontractible loops, and we have the formation of cosmic strings. If $\pi_2(M) = \pi_2(G/H) \neq 1$, M has noncontractible 2-spheres, and we have the formation of monopoles. If $\pi_3(M) = \pi_3(G/H) \neq 1$, M has noncontractible 3-spheres, and we have the formation of textures.

Once the defect is formed, the stability condition can be expressed in terms of the total transition $G \to K$. If $\pi_0(G/K) \neq 1$, domain walls are stable down to K. If $\pi_1(G/K) \neq 1$, cosmic strings are stable down to K. If $\pi_2(G/K) \neq 1$, monopoles are stable down to K. If $\pi_3(G/K) \neq 1$, textures are stable down to K.

These are conditions of topological stability of the defect. We also have dynamical stability conditions, as of what happens in collisions of strings, formation of loops, etc. We refer the reader to [4] for details about dynamical stability of defects. We may check that, in previous examples, the respective homotopy groups are nontrivial. An important result about homotopy groups is that $\pi_n(G/K) = \pi_{(n-1)}(K)$, and there is an interesting consequence of this relation for GUT phase transitions. In GUT theories a gauge group G is broken in several stages down to $K = SU(3) \otimes U(1)$. Since in this case $\pi_1(K) = \mathcal{Z}$, the integers, we have $\pi_2(G/K) \neq 1$ and therefore monopole solutions exist [11].

4 SUSY Strings

Now we turn to SUSY GUT models and how they can produce cosmic strings [5, 1, 6, 13, 12]. When we break $U(1)$, we produce cosmic strings. We assume a theory in which we have a vector superfield V and m chiral superfields Φ_i with $U(1)$ charges q_i. We work in Wess-Zumino (WZ) gauge [15], where these superfields can be written in terms of component fields as

$$V(x, \theta, \bar{\theta}) = -(\theta\sigma^\mu\bar{\theta})A_\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x), \tag{38}$$

$$\Phi_i(x, \theta, \bar{\theta}) = \phi_i(y) + \sqrt{2}\theta\psi_i(y) + \theta^2 F_i(y), \tag{39}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and θ, $\bar{\theta}$ are anticommuting coordinates of the superspace. Here ϕ_i are complex scalar fields and A_μ is a vector field, which correspond to the familiar bosonic fields of the abelian Higgs model. The fermions are $\psi_{i\alpha}$, $\bar{\lambda}_\alpha$ and λ_α. The complex bosonic field F_i and the real bosonic field D are auxiliary fields. Define the supersymmetric covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \tag{40}$$

$$\bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \tag{41}$$

in terms of which we may define the field strength chiral superfield W_α

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V. \tag{42}$$

Then, the superspace Lagrangian is given by

$$\mathcal{L} = \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta^2} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}|_{\bar{\theta}^2}\right) + \left(\bar{\Phi}_i e^{gq_iV}\Phi_i\right)|_{\theta^2\bar{\theta}^2} + W(\phi_i)|_{\bar{\theta}^2} + \kappa D. \tag{43}$$

where W is the superpotential, which is a holomorphic function of the chiral superfields (a function of only Φ_i, not $\bar{\Phi}_i$), and $W|_{\theta^2}$ indicates the θ^2 component of W. The last term, linear in D is the Fayet-Iliopoulos term [2], and can only be present in a $U(1)$ theory, since it is not invariant under more general gauge transformations.

For a renormalizable theory, the most general superpotential is

$$W(\Phi_i) = a_i\Phi_i + \frac{1}{2}b_{ij}\Phi_i\Phi_j + \frac{1}{3}\Phi_i\Phi_j\Phi_k, \tag{44}$$

This can be written in terms of the component fields as

$$W(\phi_i, \psi_j, F_k) = a_iF_i + b_{ij}\left(F_i\phi_j - \frac{1}{2}\psi_i\psi_j\right) + c_{ijk}\left(F_i\phi_j\phi_k - \psi_i\psi_j\phi_k\right). \tag{45}$$

By using Eqs.(38),(39),(42),(45) we can rewrite the Lagrangian Eq.(43) in WZ gauge in terms of the component fields. The equations of motion for the auxiliary fields F_i and D are

$$F_i^\star + a_i + b_{ij}\phi_j + c_{ijk}\phi_j\phi_k = 0 \tag{46}$$

$$D + \kappa + \frac{g}{2}q_i\bar{\phi}_i\phi_i = 0 \tag{47}$$

We can use these Eqs. to eliminate F_i and D and obtain the Lagrangian in terms of component fields

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - V \tag{48}$$

where

$$\mathcal{L}_B = (D_\mu^{i\star}\bar{\phi}_i)(D^{i\mu}\phi_i) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{49}$$

$$\mathcal{L}_F = -i\psi_i \sigma^\mu D_\mu^{i\star} \bar{\psi}_i - i\lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i, \quad (50)$$

$$\mathcal{L}_Y = \frac{ig}{\sqrt{2}} q_i \bar{\phi}_i \psi_i \lambda - \left(\frac{1}{2} b_{ij} + c_{ijk}\right) \psi_i \psi_k + (c.c.), \quad (51)$$

$$V = |F_i|^2 + \frac{1}{2} D^2 \quad (52)$$

$$= |a_i + b_{ij}\phi_j + c_{ijk}\phi_j \phi k| + \frac{1}{2}(\kappa + \frac{g}{2} q_i \bar{\phi}_i \phi_i)^2, \quad (53)$$

and

$$D_\mu^i = \partial_\mu + \frac{1}{2} i g q_i A_\mu \quad (54)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (55)$$

Before spontaneous symmetry breaking in these theories, each term in the superpotential must be gauge invariant. In the $U(1)$ case, Eq.(44) then tells us that either the coefficient of the chiral field is zero, or the charge of the term is zero. That leads to the conditions

$$\begin{aligned} a_i &\neq 0 \text{ only if } q_i = 0 \\ b_{ij} &\neq 0 \text{ only if } q_i + q_j = 0 \\ c_{ijk} &\neq 0 \text{ only if } q_i + q_j + q_k = 0 \end{aligned} \quad (56)$$

We can break the gauge symmetry by either choosing the superpotential conveniently or, in the particular case of the $U(1)$ gauge group, we may rely on a non-zero Fayet-Iliopoulos term. If we work with 3 chiral superfields, one possible choice for ther superpotential consistent with Eqs.(56) is to set $b_{ij} = 0$ and take one field to have charge zero and the other two fields charge +1 and -1. This choice will denote the F-theory. The choice of spontaneous symmetry breaking due to the non-zero Fayet-Iliopoulos term will denote the D-theory. Since the F-theory does not require a $U(1)$ gauge group, we expect it to be more representative of general defect-forming theories. For that reason we will discuss the F-theory, and refer the reader to [1] for a discussion of the D-theory.

4.1 F-theory

It turns out that the simplest possible theory with vanishing Fayet-Iliopoulos term and spontaneous break of the gauge symmetry has 3 chiral superfields [1]. Fewer fields leave the symmetry unbroken or produce anomalies. We take then two charged fields $\Phi_\pm$, with respective $U(1)$

charges $q_\pm = \pm 1$ and a neutral field ϕ_0. Setting $a_0 = -\mu\eta^2$ and $c_{0+-} = \mu/2$, with all other coefficients equal to zero, the superpotential takes the form

$$W(\Phi_i) = \mu\Phi_0(\Phi_+\Phi_- - \eta^2) \tag{57}$$

Since the scalar potential V is positive-definite, it is minimized when $V = 0$, what happens when $F_i = 0$ and $D = 0$. Here, from Eq.(53) we conclude that

$$F_i = a_0 + 2c_{0+-}(\phi_+\phi_- + \phi_0\phi_+ + \phi_0\phi_-) = 0 \tag{58}$$

$$D = \frac{1}{2}\left(\frac{g}{2}q_+|\phi_+|^2 + \frac{g}{2}q_-|\phi_-|^2\right)^2 = 0 \tag{59}$$

These conditions translate to $-\mu\eta^2 + \mu(\phi_+\phi_- + \phi_0\phi_+ + \phi_0\phi_-) = 0$ and $|\phi_+|^2 - |\phi_-|^2 = 0$ and are satisfied when $\phi_0 = 0$, $\phi_+\phi_- = \eta^2$ and $|\phi_+|^2 = |\phi_-|^2$. We may take then $\phi_\pm = \eta e^{\pm i\alpha}$, where α is some function.

At this point, we notice that we are in a situation very similar to the NONSUSY case we saw in the example of gauge symmetry in section (3). In the same spirit as in that case, we shall look for solutions with cylindrical symmetry in coordinates (r, φ, z) for the bosonic sector of the theory. We consider only bosonic fields then (i.e. set the fermions equal to zero) and write the ansatz

$$\phi_0 = 0, \tag{60}$$

$$\phi_+ = \phi_-^\star = \eta e^{in\varphi} f(r), \tag{61}$$

$$A_\mu = -\frac{2}{g}\eta\frac{a(r)}{r}\delta_\mu^\varphi, \tag{62}$$

$$F_\pm = D = 0, \tag{63}$$

$$F_0 = \mu\eta^2(f^2(r) - 1), \tag{64}$$

and the equations of motion for $\phi_\pm$ and A_μ in this ansatz produce equations for the profile functions $f(r)$ and $a(r)$

$$f'' + \frac{f'}{r} - n^2\frac{(1-a)^2}{r^2} = \mu^2\eta^2(f^2 - 1)f, \tag{65}$$

$$a'' - \frac{a'}{r} = -g^2\eta^2(1-a)f^2, \tag{66}$$

with the usual boundary conditions that assure that our solution reduce to the ground state at infinity

$$f(0) = a(0) = 0, \tag{67}$$

$$f(\infty) = a(\infty) = 1. \tag{68}$$

We notice here a very interesting point of cosmic strings in SUSY models. The ground state of the theory is supersymmetric ($V = 0$), but spontaneously breaks the gauge symmetry. On the other hand, in the core of the cosmic string, the gauge symmetry is restored ($\phi_i = 0, A_\mu = 0$)), but SUSY is spontaneously broken because $|F_i|^2 = |F_0|^2 = \mu^2\eta^4$ implies $V \neq 0$.

We have constructed here a cosmic string solution in the bosonic sector of the theory. It is also possible to do a similar analysis in the fermionic sector, setting the bosons equal to zero [1, 16]. It is possible then to conclude that there are fermion solutions that can represent superconducting currents flowing along the string at the speed of light. In these solutions, SUSY breaking is also confined to the core of the string.

5 Constraining SUSY GUTs from cosmology

An acceptable SUSY GUT model should be in agreement with both the standard model of particle physics and cosmology. For instance, the phase transitions that we mentioned in section (1) must happen, so the GUT gauge group G must be broken at the GUT scale down to the standard model gauge group $SU(3) \otimes SU(2) \otimes U(1)$ and later the electroweak transition must happen, breaking it to $SU(3) \otimes U(1)$. Also, the model should get rid of unwanted topological defects, like domain walls and monopoles. We should then either rule out models that predict their existence, or require them to be formed before (or in the early stages of) inflation. Whatever density perturbations these models produce should also be consistent with the structure in the universe, i.e. the matter power spectrum, and also the CMB anisotropies.

There have been many studies of what possible GUT models would be consistent with particle physics and cosmological observations [5, 6, 3, 13]. In a recent paper, Jeannerot et al.[6] assume a standard hybrid inflationary model and, in the context of SUSY GUTs with a F-theory, and try to find the models that solve the GUT monopole problem, leading to baryogenesis after inflation and are consistent with proton lifetime measurements. They conclude all models consistent with those assumptions produce cosmic strings.

On the other hand, current knowledge of CMB anisotropies already imposes strict constraints on the effect cosmic strings would have on CMB power spectrum [3]. These constraints will be even more strict in the near future, so we may hope to constrain even more GUT models, allowing only those consistent with CMB and other observables in the universe.

Other studies considered also D-theory supergravity models that are consistent with observations [13]. The hope is to find a model that does a perfect job: Monopoles form, inflation originates naturally from the GUT itself and solves the monopole problem, and in addition it fits with the CMB data, as well as other observations, as for instance the fact that this is the perfect moment to finish this paper.

6 Conclusion

In this paper, we discussed how strings, and more general defects naturally appear in both NONSUSY and SUSY theories. We examined the F-theory for strings in SUSY models and we mentioned studies of the constraints of GUT models that have been done from cosmological observations.

There are many important topics in the context of cosmic strings which we did not address in this paper. We did not discuss how strings actually leave their imprints on the cosmological observables, namely, the matter power spectrum, the CMB, etc. We did not discuss dynamics of strings, which has to do with their stability and evolution. We did not talk about supergravity models that produce strings. These are typically used in D-theory, since the F-theory requires only global SUSY. We also did not address the question of the formation of cosmic superstrings in the context of string theory. [10]. But with or without strings in the universe, beers will still be consumed on Friday evenings [8].

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